2015

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-103

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Ordinary Differential Equations and Special Functions*)

Answer *Q. No. 1* and any *three* from the rest.

1. Answer any *five* questions : 5×2

(a) Define fundamental set of solutions and fundamental matrix for system of differential equation.

(b) What do you mean INDICIAL equation concerning ODE ?

(c) Write the properties of Sturm Lionvilles problem.

(d) What do you mean by Wronskian in ODE and state its utility ?

(Turn Over)

(e) Examine that whether infinity is a regular singular point for Bessel's differential equation or not.

(f) Find all the singularities of the following differential equation and then classify them :

$$(z-1)\frac{d^2w}{dz^2}+(\cot\pi z)\frac{dw}{dz}+\left(\operatorname{cosec}^2\pi z\right)w=0.$$

(g) Define Green's function of the differential operator L of the non-homogeneous differential equation :

$$Lu(x) = f(x).$$

2. (a) Let $w_1(z)$ and $w_2(z)$ be two solutions of $(1-z^2)\,w''(z) - 2z\,w'(z) + (\sec z)w = 0$ with Wronskian $w(z)$. If $w_1(0) = 1$, $w_1'(0) = 0$ and $w\left(\frac{1}{2}\right)=\frac{1}{3}$, then find the value of $w_2'(z)$ at $z = 0$. 4

(b) Using Green's function method, solve the following differential equation :

$$y^{IV}(x) = 1$$

subject to boundary conditions

$$y(0) = y'(0) = y''(0) = y'''(1) = 0.$$ 6

3. (a) Discuss the solution procedure for solving the homogeneous vector differential equation in the form

$$\frac{dx}{dt} = Ax, \text{ where } x=\begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix} \text{ and } A=(a_{ij})_{n\times n}$$

matrix (where the eigen values are real distinct). 6

(b) Prove that :

$$\int_{-1}^{1} P_m(z)\, P_n(z)\, dz = \frac{2}{2n+1}\delta_{mn}$$

where δ_{mn} and $P_n(z)$ are the Knoneker delta and Legendre's polynomial respectively. 4

4. (a) Find the eigen values and eigen functions of the Storm-Liouville system :

$$\frac{d^2y}{dx^2} + \lambda y = 0, \ 0 < x < \pi,$$

and the boundary conditions are

$$y(0) = 0, \ y'(\pi) = 0.$$ 4

(b) Deduce Rodrigue's formula for Legendre's polynomial. 4

(c) Establish the integral representation of confluent hypergeometric function. 2

5. (a) Establish the generating function for Bessel's function $J_n(z)$. Use it, prove the following :

$$Z J_n'(z) = Z J_{n-1}(z) - n' J_n(z).$$ 5

(b) Show that eigen functions of a regular Sturm-Liouville system :

$$\frac{d}{dx}\left\{p(x)\frac{du}{dx}\right\} + \{\lambda\ p(x) - q(x)\}\, u = 0,$$

where λ is a parameter, p, ρ and q are real-valued functions of x, p and ρ being positive, having different eigen values are orthogonal with respect to the weight function $p(x)$. 5

6. (a) Show that :

$$w(z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k!\,(c)_k} z^k$$

where $(a_k) = a(a+1)\ldots(a+k-1)$ etc. is a solution of the hypergeometric equation :

$$z(1-z)\frac{d^2w}{dz^2} + \{c-(a+b+1)z\}\frac{dw}{dz} - abw = 0.$$ 6

(b) Let $P_n(z)$ be the Legendre polynomial of degree n such that $P_n(1) = 1,\ n = 1, 2, 3, \ldots$

If $\int_{-1}^{1}\left(\sum_{j=1}^{2}\sqrt{j(2j-1)}\,P_j(z)\right)^2 dz = 20$, then find the value of n. 4

(Internal Assessment — 10 Marks)