2015

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-102

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Complex Analysis*)

Answer *Q. No. 1* and any *eight* from the rest.

1. Answer any *four* questions : 4×2

(*Write only the appropriate answers by preparing a table. Do the rough works after the table.*)

(i) The square roots of (5−12i) are ....

(ii) All the points of discontinuity of the function $f(Z)=\frac{\tanh Z}{Z^2+1}$ are ....

(Turn Over)

(iii) Is the function $u = 2xy + 3xy^2 - 2y^3$ harmonic ?

(iv) The principal value of $(-1)^{\log(1+i)}$ is ....

(v) Res f(z) at z = 0 where

$$f(z) = \frac{z-3}{z^2}\sin\frac{1}{1-z} \text{ is } \ldots\ldots\ldots$$

(vi) The value of $\oint_c \frac{\cos^2 tz}{z^3} dz$ where c is the circle $|z| = 1$ and $t > 0$, is ....

2. Determine the analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$. 4

3. If a function f(z) is continuous on a contour c of length l and if M be the upper bound of $|f(z)|$ on c, then prove that :

$$\left|\int_c f(z)\, dz\right| \leq Ml.$$ 4

4. Find the singular points of the following function, if any, $f(z) = z \cdot |z|$. Justify. 4

5. Find the Laurent series that represent the function $\frac{1}{z^3-5z^2+8z-4}$ in the following domains (i) $0 < |z-1| < 1$ and (ii) $0 < |z-2| < 1$. 4

6. Find the residue of $\frac{1}{z^3-5z^2+8z-4}$ at its poles inside $c : |z| = 3$ and hence evaluate $\int_c \frac{1}{z^3-5z^2+8z-4} dz$, in counter clockwise sense.

7. Using the method of residues, evaluate (*no marks will be awarded if method of residues is not used*) :

$$\int_{-\infty}^{\infty} \frac{dx}{\left(x^2+4\right)^3}.$$ 4

8. Using the method of residues, evaluate (*no marks will be awarded if method of residues is not used*) :

$$\int_0^{2\pi} \frac{1+\sin\theta}{2+\cos\theta} d\theta.$$ 4

9. Evaluate the integral $\int_c \frac{\sinh \pi z}{(z-1)\left(z^2+1\right)} dz$ where c is the simple closed contour $|z-1-i| = \frac{5}{2}$, in counter clockwise sense. 4

10. Show that the bilinear transformation $w = \frac{az+b}{cz+d}$ transforms the circle

$$\arg\left(\frac{z-z_1}{z-z_2}\right) = \lambda$$

into similar circle $\arg\left(\frac{w-w_1}{w-w_2}\right)$ = constant

where w_1 and w_2 correspond to z_1 and z_2, respectively. 4

11. Prove that the poles of an analytic function are isolated.

Let $f(z) = u + iv$ be analytic in a region.

Show that :

$$\frac{\partial(u,v)}{\partial(x, y)} = | f'(z) |^2 .$$ 4

12. Use Rouche's theorem to find the number of zero of

$$z^{10} + a_1z^4 + a_2z^3 + a_3z^2 + a_4z + a_5 = 0$$

in $|z| = 1$ if

$$|a_1| > |a_2| + |a_3| + |a_4| + |a_5| + 1.$$ 4

13. Find the Möbius transformation that maps the points $z_1 = \infty$, $z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0$, $w_2 = i$ and $w_3 = \infty$, respectively. 4

(Internal Assessment — 10 Marks)

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