PG/IVS/MTM-405/15

M.Sc. 4th Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Special Paper : *Dynamical Meteorology* - II/ *Operational Research Modeling* - II)

PAPER — MTM - 405

Unit – I

Full Marks : 25

Time : 1 hour

The figures in the right-hand margin indicate marks

(*Dynamical Meteorology* - II)

[*Marks* : 25]

Time : 1 hour

Answer **Q.No. 1** and any **two** from the rest

1. Answer any *one* questions : 2 × 1

(*a*) What are the dynamic and kinematic boundary conditions at a front ?

(*Turn Over*)

(*b*) Write down the difference between stream line and turbulent motions in the atmosphere.

2. What is Global circulation in the atmosphere ? Derive the meridional temperature gradient. How Jet stream and Rossby wave are developed ? Hence derive the expression at the parameter β.
1 + 3 + 2 + 2 + 1

3. (*a*) Derive the pressure tendency below a frontal surface. 5

(*b*) Derive the necessary condition for frontogenesis or frontolysis. 4

4. (*a*) Show that in a geostrophic wind field, an ideal front is necessarily stationary. 2

(*b*) Derive the equation to determine the diffusion of water vapor through the atmosphere by turbulent mixing process. 7

[*Internal Assessment* : 5 Marks]

(*Operational Research Modeling* - II)

[*Marks* : 25]

Time : 1 hour

Answer **Q.No. 1** and any **two** from the rest

1. Answer any *two* questions : 2 × 2

(*a*) A particle is attached to the lower end of a vertical spring whose other end is fixed, is oscillating about its equilibrium position. The governing differential equation of motion is $\ddot{x} = -w^2 x$. If a force u per unit mass is added to the lower end, then its equation of motion becomes $\ddot{x} = -w^2 x + u$, where x and u are the displacement from equilibrium position and control variable. Explain the role of u about the motion of the particle. Why u is called control variable ?

(*b*) What do you mean by memory less channel and noiseless channel ?

(*c*) State principal assumptions made on sequencing problem.

2. (*a*) In a system, there are n number of components connected in parallel with reliability $R_i(t)$, $i = 1, 2, ..., n$. Find the reliability of the system. IF $R_1(t) = R_2(t) = ... = R_n(t) = e^{-\lambda t}$, then find the expression for the system reliability. 3

(*b*) How many identical component each of which is 90% reliable over a period of 50 hours be used to obtain a 99·99% parallel redundancy system over 50 hours. If we want to obtain the same system reliability over a period of 100 hours, how many components should be added ? 3

(*c*) Show that MTBF of the system of n identical units connected parallelly is

$$\frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i}$$

where λ is the failure rate of each component. 2

3. (*a*) Find the sequence that minimizes the total elapsed time and ideal time required for all three machines to complete the following tasks : 5

Tasks	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Time on I machine	3	8	7	4	9	8	7
Time on II Machine	4	3	2	5	1	4	3
Time on III Machine	6	7	5	11	5	6	12

(*b*) Prove that the reliability function for random failures is an exponential distribution. How system reliability can be improved ? 3

4. Suppose a car is derived from a stationary position on a horizontal way to a stationary position in a garage moving a total distance a. The available control for the driver are the accelerator and the break (for simplicity we consider no gear change). The lower and upper bounds of the control variable f (acceleration or deceleration) are α and β. Show that the minimum

time to bring the car in the stationary position at a distance a is $\sqrt{\frac{2a(\alpha+\beta)}{\alpha\beta}}$, the optimal control to be applied on the car is given by

$$f = \begin{cases} \beta, & 0 \le t \le \tau \\ -\alpha, & \tau \le t \le T \end{cases}$$

Where $\tau = \sqrt{\frac{2a\alpha}{\beta(\alpha+\beta)}}$. 8

[*Internal Assessment* : 5 Marks]
