M.Sc. 4th Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Special Paper : *Non-linear Optimization/ Dynamical Oceanology* - II)

PAPER – MTM - 404(OR/OM)

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

MTM – 404(OR)

(*Non-linear Optimization*)

Answer **Q.No. 1** and any **three** from the rest

1. Answer any *five* from the following : 2 × 5

(*a*) What is the necessity of constaint qualification related with non-linear programming ?

(Turn Over)

(*b*) How is the degree of difficulty defined for a geometric programming problem ? Give an example of geometric programming problem which has negative degree of difficulty.

(*c*) What is stochastic programming problem ? Give an example of stochastic programming problem.

(*d*) Define Nash equilibrium solution and Nash equilibrium outcome in mixed strategy for bimatrix game.

(*e*) What is multi-objective non-linear programming problem ? Give an example of it.

(*f*) State Kuhn-Tucker stationary-point necessary optimality theorem.

(*g*) What do you mean by complementary stackness conditions concerning on Wolfe's method.

2. (*a*) What do you mean by quadratic programming problem ? Derive Kuhn Tucker conditions for quadratic programmings problem. Under what conditions, the above Kuhn-Tucker condition will be necessary and sufficient ? 1 + 5 + 1

(*b*) Prove that all strategically equivalent bimatrix games have the same Nash equilibria. 3

3. (*a*) State and prove fritz John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP. 7

(*b*) What is differential convex function ? Give the geometrical interpretation of it. 3

4. (*a*) Find $x_1 > 0$, $x_2 > 0$ and $x_3 > 0$ that minimizes

$$f(x_1, x_2, x_3) = x_1 x_2 x_3^{-2} + 2x_1^{-1}x_2^{-1}x_3 + 5x_2 + 3x_1 x_2^{-1}$$ 5

(*b*) Write the relationship among the solutions of local minimization problem (LMP), the minimization problem(MP), the Fritz John stationary problem (FJP), the Fritz. John saddle point problem (FJSP) the Kuhn-Tucker stationary point problem (KTP) and the kuhn-Tucker saddle point problem (KTSP). 5

5. (*a*) Solve the following problem by Beale's method

$$\text{Max } Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

subject to the constraints

$$x_1 + 2x_2 \leq 2$$
$$x_1, x_2 \geq 0$$ 6

(*b*) State and prove slater's theorem of the alternative. 4

6. (*a*) State Forkas' theorem of nonlinear programming and give the geometrical enterpretation of it. 5

(*b*) Define the following terms :

(*i*) The (primal) quadratic minimization problem (QMP).

(*ii*) The quadratic dual (maximization) problem (QDP). 2

(*c*) State and prove weak duality theorem related to duality in quadratic programming. 3

[*Internal Assessment* : 10 Marks]

MTM – 404(OM)

(*Dynamical Oceanology* - II)

Answer any **four** questions

1. Mention the assumptions of Ekman model. Show that the total flow, under Ekman model, along the shoreline is give by

$$S = \frac{\tau}{if}\left(1 - \frac{1}{\cos h\,\mu H}\right) - \frac{gP}{i\mu f}(\mu H - \tan h\,\mu H),$$

(symbols have their usual meanings). 2 + 8

2. Prove that the equation of western boundary current can be expressed as

$$\Omega E\sqrt{\sin\phi}\,\nabla_h^2\psi + \frac{2\Omega}{a^2}\frac{\partial\psi}{\partial\lambda} = rot_z(\tau)$$

(with usual symbols). 10

3. Show that in the neighbourhood of the eastern shore, the viscous boundary layer in two dimensional model of ocean currents can be written as $\psi = O(\delta)$; and also find $\frac{\partial\psi}{\partial x_2}$ where x_2 is curvilinear orthogonal coordinate system of eastern shore. 8 + 2

4. Show that the perturbation temperature T_s outside of the western boundary layer for the linear model of thermocline may be expressed as

$$T_s = \frac{2\theta(y)}{\pi}\int_0^\infty \frac{\left(1-e^{-x^2 f^2 \tau^4}\right)}{\tau}\sin x\tau\, d\tau,$$

(symbols have their usual meanings). 10

5. (*a*) Derive the equations of inertial flow. 6

(*b*) Discuss the nature of the inertial flow. 4

6. (*a*) Define Rossby number. Derive its mathematical expression. 7

(*b*) Give its physical interpretations. 3

[*Internal Assessment* : 10 Marks]