2015

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-101

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Real Analysis*)

Answer *Q. No. 1* and any *four* from Q. No. 2 to Q. No. 7.

1. Answer any *four* questions : 4×2

(a) Which of the following are connected ?

(i) $\{(x, y) \in \mathbb{R}^2 : xy \neq 0\}$

(ii) $\{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R} \text{ and } y \notin \mathbb{R}\}$

Justify your answer.

(Turn Over)

(b) Show that any function from a discrete metric space into a metric space is continuous.

(c) Let I be an interval of $\mathbb{R}$ having at least two distinct points. Show that I is not a null set.

(d) Let $f : [a, b] \to \mathbb{R}$ be bounded on [a, b] and α be strictly monotonically increasing on [a, b]. Then show that

$$\overline{\int_a^b} f d\alpha \text{ and } \underline{\int_a^b} f d\alpha \text{ exists.}$$

(e) Define the following with example :

(i) Null sets ;

(ii) Separable metric space.

2. (a) Show that the open interval (2, 3) of $\mathbb{R}$ is not a compact subset of $\mathbb{R}$ (without using Heine-Borel theorem). 4

(b) Show that every path-connected metric space is connected. Is the converse true? 4

3. (a) Establish necessary and sufficient conditions for a function $f : [a, b] \to \mathbb{R}$ to be a function of bounded variation on [a, b]. 5

(b) Define variation function.

Hence find the variation function for the function :

$f(x) = [x] - x, x \in [1, 3]$. 3

4. (a) Define the following : Measurable sets and Measurable functions, Cantor set, Lebesgne integral for unbounded function. 5

(b) Let μ be a positive measure on a σ-algebra m.

Then show that $\mu(A_n) \to \mu(A)$ as $n \to \infty$ if $A = \bigcup_{n=1}^{\infty} A_n$,

$A_n \in m$, and $A_1 \subset A_2 \subset A_3 \subset \ldots\ldots$ 3

5. (a) If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a, b], then show that $f_1 + f_2 \in R(\alpha)$ and

$$\int_a^b (f_1 + f_2)\,d\alpha = \int_a^b f_1\,d\alpha + \int_a^b f_2\,d\alpha.$$ 5

(b) Evaluate the R–S integral : 3

$$\int_2^5 \left(5x^5 + 7e^{5x} - 4x^3 + 3x + 2\right) d\left(3[x] + 1\right).$$

6. (a) Let f be a bounded function and Lebesgue integrable on [a, b]. Also, let g be a bounded function on [a, b] such that f(x) = g(x) a.e. on [a, b]. Then show that g is also Lebesgue integrable on [a, b] and

$$\int_a^b f(x)\,dx = \int_a^b g(x)\,dx.$$ 4

(b) Let f(x) be defined on [0, 1] where

$$f(x) = \begin{cases} \frac{1}{x^{\frac{3}{4}}}, & 0 < x < 1 \\ 0, & x = 0 \end{cases}$$

Check whether f is Lebesgue integrable on [0, 1]. If it is Lebesgue integrable, then find the value of the integral. 4

(Internal Assessment — 10 Marks)