2015

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-305

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

(Dynamical Meteorology-I / Operational Research Modelling-I)

(Operational Research Modelling — I)

Answer *Q. No. 1* and any *four* from the rest.

1. Answer any *four* questions of the following: 4×2

(a) State Bellman's principle of Optimality in connection with dynamic programming problem.

(Turn Over)

(b) What are the advantages of network analysis ?

(c) Explain Monte-Carlo simulation.

(d) Explain the terms :

'Critical activities' and 'Critical path'.

(e) Discuss 'gradual failure' and 'sudden failure' of items with examples.

(f) What are the objectives of inventory control ?

2. Generate 10 random numbers using the formula $x_{n+1} \equiv (a\, x_n)(\text{mod } 100)$ starting from $x_0 = 357$ and $a = 273$.

Use these random numbers for solving the following problem :

A bakery keeps stock of a popular band of Cake. Previous experience shows the daily demand pattern for the items with associated probabilities, as given below :

Daily demand :	0	10	20	30	40	50	60
Probability :	0·01	0·10	0·25	0·40	0·15	0·05	0·04

Simulate the demand for next 10 days. Also estimate the daily average demand for the cakes.

8

3. Derive the differential difference equations for (M|M|1) : (N|FCFS|∞) queueing model for transient state. Also, find the probability (p_n) of n customer who is in the system. 8

4. Use dynamic programming technique, to show that

$$\sum_{i=1}^{n} p_i \log p_i, \text{ subject to } \sum_{i=1}^{n} p_i = 1, \text{ is minimum when}$$

$$p_1 = p_2 = \ldots = p_n = \frac{1}{n}.$$ 8

5. Derive the optimal ordering policy for a single period continuous probabilistic model without set-up cost. 8

6. A shopkeeper estimates that the annual requirement of an item is 2,000 units. He buys it from his supplier at a cost of Rs. 10 per item and the cost of ordering is Rs. 50 each time he orders. If the stock holding costs are 25 per cent per year of stock value, how frequently should be replenish his stock ?

Further, suppose the supplier offers a 10 per cent discount on orders between 400 and 699 items, and a 20 per cent discount on orders exceeding or equal to 700.

Can the shopkeeper reduce his costs by taking advantage of either of these discount ? 8

7. A research and development department is developing a new power supply for a console television set. It has broken the job down into the following :

Job	*Description*	*Immediate predecessors*	*Time* (Days)
A	Determine output voltages	—	5
B	Determine whether to use solid state rectifiers	A	7
C	Choose rectifier	B	2
D	Choose filters	B	3
E	Choose transformer	C	1
F	Choose chassis	D	2
G	Choose rectifier mounting	C	1
H	Layout chassis	E, F	3
I	Build and test	G, H	10

(a) Draw a network diagram for the project.

(b) Find the critical path. What is its length ?

(c) Find the total float and free float for each non-critical activities.

2+3+3

(Internal Assessment — 10 Marks)

(Dynamical Meteorology — I)

Answer *Q. No. 8* and any *four* from the rest.

1. (a) Deduce the equation of state for moist air in the following form :

$$p\alpha = \frac{R^*}{m_d}\left(\frac{1+\frac{w}{\epsilon}}{1+w}\right)T$$

and show that the virtual temperature is always higher than the actual temperature. 5+2

(b) Derive the dry adiabatic lapse rate in the atmosphere. 2

2. (a) How is the thermal wind formed in the atmosphere ? Derive the thermal wind components in the atmosphere. 2+5

(b) Derive the expression of vorticity in the atmosphere in terms of natural co-ordinate. 2

3. (a) Derive the relation by which the local rate of temperature change be occured. Hence show that the descending or ascending motion results is local warming or cooling provided that $\gamma < \gamma_d$. 5

(b) Prove that $T_V = T(1 + 0.61r)$ where T_V be the virtual temperature, T be temperature and r be the mixing ratio of an air parcel. 2

(c) Show that in an isothermal atmosphere, the pressure decreases exponentially with height. 2

4. (a) Discuss the Slice method of stability Analysis in the atmosphere. 7

   (b) Find the relation between mixing ratio and specific humidity. 2

5. (a) Derive the vertical shear of a geostrophic wind. Hence show that in a barotrophic atmosphere, the geostrophic wind is constant with respect to height. 6

   (b) After giving the definition of potential temperature, show that it is invariant in the atmosphere. 3

6. (a) State and prove the moisture conservation equation in the atmosphere. 7

   (b) What do you mean by equivalent temperature ? 2

7. (a) Derive the equation of horizontal motion of an air parcel in quasi-Lagrangian co-ordinates. 4

   (b) Derive the adiabatic lapse rate for moist saturated air. 5

8. Answer any *two* questions : 2×2

   (a) Explain geo-dynamical paradox.

   (b) What is wet bulb temperature ?

   (c) What are the characteristics of aerological diagram ?

(Internal Assessment — 10 Marks)