2015

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-304

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

(*Dynamical Oceanology-I/Advanced Optimisation and Operations Research*)

(Advanced Optimization and Operations Research)

Answer *Q. No. 1* and any *four* from the rest.

1. Answer any *four* questions of the following: 4×2

(a) State the sufficient conditions for multivariate optimization with inequality constraints. 2

(b) What are deviational variables? How do they differ from decision variables in traditional linear programming problem? 1+1

(c) Define 'conjugate directions' and 'quadratically convergent method'. 1+1

(d) Discuss the effect of deletion of a variable from the final optimal table of simplex method. 2

(e) What is basic difference between Fibonacci method and Golden section method ? Which one is better and why ? 1+1

(f) Compare revised simplex method with dual simple method.

2. State the following integer programming problem using branch and bound method : 8

Maximize $Z = 7x_1 + 6x_2$

subject to $2x_1 + 3x_2 \leq 12$

$6x_1 + 5x_2 \leq 30$

$x_1, x_2 \geq 0$ and integers.

3. Let A be a real symmetric square matrix of order n and the quadratic function $Q(X)=\frac{1}{2}X^TAX+B^TX+C$ is minimized sequentially once along each direction of a set of n A-conjugate directions. Show that the global minimum of Q(X) will be located at a before the nth step regardless of the starting point and the order in which the directions are used.

4. What is goal programming?

A shoe manufacturer produces hiking boots and ski boots. the manufacturing process of the boots consists of sewing and stitching. The company has available 60 hours per week for the sewing process and 80 hours per week for the stitching process at normal capacity. The firm realizes profits of Rs. 150 per pair on hiking boots and Rs. 100 per pair on ski boots. It requires 2 hours of sewing and 5 hours of stitching to produce one pair of hiking and 3 hours of sewing and 2 hours of stitching to produce one pair of ski boots. The president of the company wishes to achieve the following goals, listed in the order of their importance :

(i) Achieve the profit goal Rs. 5,250 per week.

(ii) Limit the overtime operation of the sewing center to 30 hours.

(iii) Meet the sales goal for each type of boot 25 hiking boots and 20 ski boots.

(iv) Avoid any under utilization of regular operation hours of the sewing center.

Formulate this problem as a goal programming model. 2+6

5. Use the revised simplex method to solve the following linear programming problem :

$$\text{Maximize } z = 6x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_3 \leq 4$$

$$\text{and } x_1, x_2, x_3 \geq 0. \qquad 8$$

6. What are the limitations of Fibonacci method ? Write down the procedure of Fibonacci search method to find the interval of uncertainty of the maximization problem after n experiments.

7. The optimal solution of the following LP problem :

$$\text{Maximize } z = 3x_1 + 5x_2$$

$$\text{subject to } 3x_1 + 2x_2 \leq 18$$

$$x_1 + 2x_2 \leq 4$$

$$x_2 \leq 6$$

$$\text{and } x_1, x_2 \geq 0$$

is contained in the following table :

		c_j	3	5	0	0	0
c_B	x_B	b	y_1	y_2	y_3	y_4	y_5
3	x_1	2	1	0	1/3	0	-2/3
0	x_4	0	0	0	-2/3	1	4/3
5	x_2	6	0	1	0	0	1
$z_j - c_j$			0	0	1	0	3

(i) Discuss the effect on the optimal basic feasible solution by adding a new constraint $2x_1 + x_2 = 8$ to the given set of constraints. 8

(Internal Assessment — 10 Marks)

(Dynamical Oceanology – I)

Answer *Q. No. 1* and any *four* from the rest.

1. Answer any *four* questions of the following: 4×2

(a) What is the source of forces (acting on a moving fluid element) involved in Newton's second law, for the derivation of momentum equation?

(b) Write the equation of Navier Stokes, component wise, for the case of coriolis, gravity, frictional and tidal forces.

(c) Draw the vertical structure of ocean showing the three layers : Mixed, Thrmodaline and Deep Ocean.

(d) Write the certain state of variables involved in the thermodynamical state of a single homogeneous chemically inert gas of a constant mass.

(e) Write the Gibb's relation for basic thermodynamic parameters (T, μ_j and p) as a function $\eta(E, \tau, C_j)$.

(f) What is salinity ? Define the salinity for $(n-1)$ components salt and water.

2. (a) Draw an infinitesimally small moving fluid element.

(b) Show all the forces (involved for the derivation of y-component of the montum equation) in the y-direction in the above fluid element.

(c) Finally derive the y-momentum equation in non-conservation form.

1+3+4

3. (a) Write the x-component of Navier-Stokes equation in non-conservation form.

(b) Convert the above equation to its conservation form.

(c) Using the expression for shear and normal stress obtained by Stokes, simplify the above equation, which is free from second viscosity coefficient.

1+2+5

4. Show the energy fluxes (in the χ direction) associated with an infinitesimally small moving fluid element for the derivation of the energy equation.

Hence, derive the energy equation in non conservative form. 3+5

5. (a) Define the coefficient of thermal expansion (volumetry) : α, isothermal compression : K_T and adiabatic compression : K_η in terms of τ, p and J.

(b) Derive the relation :

$$K_\eta = K_T - \frac{T}{\tau C_p}.\left(\frac{\partial \tau}{\partial T}\right)^2$$

(c) For $K_\eta = K_T - \Gamma_\alpha$,

deduce that $K_T > K_\eta$.

3+3+2

6. Derive the equation of motion of a particle referred to a frame of reference rotating with the Earth by considering the centripetal and coriolis forces. 8

7. Assuming that the mass exchange processes across the free ocean surface, $F(\bar{r}, t) = 0$, amount to a flux 'b' of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface. 8

(Internal Assessment — 10 Marks)