# 2015

## M.Sc.

## 3rd Semester Examination

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

### PAPER—MTM-302

*Full Marks* : 50

*Time* : 2 Hours

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**(*Transform and Integral Equation*)**

Answer *Q. No. 1* and any *three* from the rest.

1. Answer any *five* questions of the following: 2×5

(a) Find the Laplace transform of the function f(x) = [x], where [x] represents the greatest integer less than or equal to x.

(b) When a function f(x) is said to be of exponential order $O(e^{ax})$ for $x > 0$ ? If f(x) is of exponential order, what extra condition is needed for the existence of its Laplace transform ?

*(Turn Over)*

(c) Who first coined the term Wavelet ? Define 'mother wavelet' and explain the utility of it.

(d) What do you mean by Singular integral equation ? Define degenerate Kernal with an example.

(e) What do you mean by an inverse Fourier transform of a specified function ?

(f) Define the term 'FALTUNG' concerning on Fourier transform.

(g) If the function f(t) has period $T > 0$ then calculate $L\{f(t)\}$ in a simplest form.

**2.** (a) State Parserval's identity on Fourier transform.

Use generalization of Parseval's relation to show that

$$\int_{-\infty}^{\infty} \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)} = \frac{\pi}{ab(a+b)},\ a, b > 0.$$ 1+3

(b) Find the resolvent Kernel of the following integral equation and then solve it :

$$\varphi(x) = e^{x^2} + \int_0^x e^{x^2-t^2} \phi(t)\, dt$$ 6

**3.** (a) Use the Laplace transformation technique to solve the differential equation :

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 2t - e^{-t},\ t > 0$$

which satisfies $x(0) = \frac{1}{2},\ \frac{dx}{dt} = 0,$ at $t > 0$. 5

(b) Define the continuous wavelet function and also explain the inverse wavelet transform. Write some important applications of wavelets. 5

4. (a) Show that if a function f(x) defined on $(-\infty, \infty)$ and its Fourier transform $F(\xi)$ are both real, then f(x) is even. Also show that if f(x) is real and its Fourier transform $F(\xi)$ is purely imaginary, then f(x) is odd. 4

(b) Solve the integral equation :

$$y(x) = f(x) + \lambda \int_0^1 (x+t)\, y(t)\, dt$$

and find the eigen values. 6

5. (a) (i) State and prove final value theorem concerning on Laplace transform.

(ii) Show that $\int_0^t \int_0^v \int_0^w f(u)\, du dv dw = L^{-1}\left\{\frac{F(p)}{p^3}\right\}$

where $F(p) = L\{f(t)\}$. 2+3

(b) Use Laplace transform to solve the integral equation :

$$\int_0^x \cos(x-t)\, \varphi(t)\, dt = x^2 - x.$$ 5

6. (a) Consider the boundary value problem :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad -\infty < x < \infty, \ y > 0,$$

satisfying the boundary conditions

$$u(x, 0) = f(x), \ -\infty < x < \infty,$$

$$u(x, y) \to 0, \text{ as } y \to \infty,$$

$$u(x, y), \ u_x(x, y) \to 0 \text{ as } |x| \to \infty.$$

Use Fourier transform to show that this solution of the boundary value problem is :

$$u(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi)\, d\xi}{(x-\xi)^2 + y^2}$$ 7

(b) State Fredholm Alternative concerning on integral equation. 3

**(Internal Assessment — 10 Marks)**