M.Sc. 2nd Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(General Topology & Fuzzy Sets and Their Applications)

PAPER – MTM - 205

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

UNIT – I

(General Topology)

[*Marks* : 25]

Answer **Q.No. 1** and any **two** from the rest

1. Answer any *two* questions : 2×2

   (*a*) Define locally connected spaces. Give an example of a space which is locally connected but not connected.

(*Turn Over*)

(b) Consider the set $Y = [-1, 1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$?

$$A = \left\{x : \frac{1}{2} < |x| < 1\right\},$$

$$B = \left\{x : \frac{1}{2} < |x| \leq 1\right\}.$$

(c) Give an example of a mapping which is continuous but not open.

2. (a) Let X be a set and B be a basis for X. Show that B generates a topology for X. 4

(b) Let A, B denote subsets of a space X. Determine whether the following equations hold. If an equality fails, determine whether one of the inclusions $\supset$ or $\subset$ holds. 4

(i) $\overline{A \cap B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A - B} = \overline{A} - \overline{B}$.

3. (*a*) Consider the product, uniform, and box topologies on $\mathbb{R}^{w}$. In which topologies does the following function f form $\mathbb{R}$ to $\mathbb{R}^{w}$ continuous ? 4

$$f(t)=\left(t,\frac{1}{2}t,\frac{1}{3}t,\ldots\right).$$

(*b*) Prove that every finite set is closed in a T_2 space. Define a $T3\frac{1}{2}$ space. 3 + 1

4. (*a*) Show that every Compact Hausdorff space is normal. 4

(*b*) State the following theorems – Urysohn metrization theorem, Tietz extension theorem, Tychonoff theorem. 4

[*Internal Assessment* : 5 Marks]

UNIT – II

(*Fuzzy Sets and Their Applications*)

[*Marks* : 25]

Answer **Q.No. 1** and any **three** from the rest

1. Answer any *two* questions : 1 × 2

   (*a*) Write the membership function of a convex fuzzy set.

   (*b*) Is fuzzy sets satisfy all the properties of crisp sets ? Justify your answer .

   (*c*) When a fuzzy set becomes a fuzzy number ?

2. State the Zadeh's extension principle of fuzzy sets. Using this show that

$$[2, 4] + [5, 7] = [7, 11].$$

2 + 4

3. (*a*) If $\tilde{A}$ is the triangular fuzzy number [1, 3, 4] determine $f(\tilde{A})$ where $f(x) = 2x + 3$. 2

   (*b*) What are causes of uncertainty ? Is there any relation between probability and fuzzy ? Justify your answer. 2 + 2

4. Illustrate Bellman and Zadeh's principle to optimize a fuzzy LPP. Discuss Werner's method to slove a fuzzy LPP. 2 + 4

5. (*a*) For any two triangular fuzzy numbers $\tilde{A} = [a_1, b_1, c_1]$ and $\tilde{B} = [a_2, b_2, c_2]$, using α-cuts show that

$$\tilde{A} + \tilde{B} = [a_1 + a_2, b_1 + b_2, c_1 + c_2].$$ 5

(*b*) What is a support of a fuzzy set ? 1

6. Solve the following fuzzy LPP by using Verdgey's method

Max. $Z = x_1 - 3x_3$

Sub. to

$$x_1 - 2x_2 - x_3 \le 1 \text{ to } 1\cdot5$$
$$-x_1 + 4x_2 - x_3 \le 2 \text{ to } 4$$
$$-x_1 - 3x_2 + x_3 \le 3 \text{ to } 5$$
$$x_1,\ x_2,\ x_3 \ge 0.$$ 6

[*Internal Assessment* : 5 Marks]