M.Sc. 2nd Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(General Theory of Continuum Mechanics)

PAPER — MTM - 204

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **four** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* questions : 4 × 2

(*a*) The strain tensor at a point is given by

$$[E_{ij}] = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Determine the extension of a line element in the direction of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$. Find also the change of angle between two perpendicular line elements in the directions $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}\right)$.

(*b*) Defining complex potential, find the complex potential for a doublet.

(*c*) Differentiate between (*i*) stream line and path line (*ii*) source and sink.

2. Derive Bernoulli's equation of motion in its most general form. Deduce Bernoulli's equation (*i*) when the fluid is incompressible and (*ii*) when the motion is steady. 6 + 1 + 1

3. (*a*) Two sources, each of strength m, are placed at the points $(-a, 0)$ and $(a, 0)$ and a sink of

strength $2m$ is placed at the origin. Show that the stream lines are curves

$$\left(x^2+y^2\right)^2 = a^2\left(x^2-y^2+\lambda xy\right)$$

where λ is a parameter. Also find the fluid speed at any point. 6

(*b*) Find the stress vector at a point on the plane whose normal vector is $\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$ and state of stress at a point is given by

$$\begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 0 \\ 3 & 0 & -1 \end{pmatrix}. \quad 2$$

4. Define principal strains and principal directions of strain. Prove that all principal strains are real and its directions of strain are orthogonal. 2 + 3 + 3

5. (*a*) The displacement in an elastic solid is given as follows :

$$u_1 = \in(x_1 + 2x_2 + 3x_3)$$