PG/IIS/MTM-202/15

M.Sc. 2nd Semester Examination, 2015

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Analysis*)

PAPER – MTM - 202

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) Prove that

$$\left(1+\delta^2\mu^2\right)f(x)=\left(1+\frac{\delta^2}{2}\right)^2 f(x),$$

where the symbols have their usual meanings.

(*b*) What is meant by ill-conditioned system of Linear algebraic equations ?

(*c*) What are the similarities and dissimilarities between modified Euler's method and second order Runge-Kutta method ?

(*d*) Compare direct and iterative method to solve a system of linear equations.

(*e*) What is the advantage to use the Tchebyshev Polynomials to approximate a function ?

(*f*) Obtain the approximate quadrature formula

$$\int_{-1}^{1} f(x)dx = \frac{1}{9}\left[5f\left(-\sqrt{0\cdot 6}\right)+8f(0)+5f\left(\sqrt{0\cdot 6}\right)\right]$$

2. (*a*) Explain Milne's predictor-corrector formula to solve the differential equation

$$\frac{dy}{dx} = f(x,y),\ \ y(x_0) = y_0. \qquad 8$$

(*b*) Explain Runge-kutta fourth order method to solve the following system of differential equations

$$\frac{dy}{dx} = f(x, y, z), \quad \frac{dz}{dx} = g(x, y, z)$$

with $y(x_0) = y_0$ and $z(x_0) = z_0$. 2

(*c*) Use Newton-Raphson method to find a root of the system $x^2 - 2x - y + 0{\cdot}5 = 0$, $x^2 + 4y^2 - 4 = 0$ with the starting value $(x_0, y_0) = (2{\cdot}0, 0{\cdot}25)$, correct up to four decimal places. 6

3. (*a*) Explain finite difference method to solve the following equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1.$$

with initial conditions

$$u(x,0) = f(x), \; u_t(x,0) = g(x), \; 0 < x < 1$$

and boundary conditions

$$u(0, t) = \phi(t); \quad u(1, t) = \psi(t), \; t \geq 0.$$ 8

(*b*) Suppose

$$y = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \frac{x^4}{8!} - \cdots$$

Economize this series if the fourth decimal place is not to be affected, near $x = 1$. 4

(*c*) Explain partial pivoting method to find the inverse of a square non-singular matrix. 4

4. (*a*) Deduce three-point Gauss-Legendre quadrature formula. 4

(*b*) Discuss the stability of second order Runge-Kutta method. 4

(*c*) Describe power method to find the largest (in magnitude) eigen-value and the corresponding eigen-vector of a matrix. Can this method be used to find the least eigen-value ? Explain. 6 + 2

[*Internal Assessment* : 10 Marks]