PG/IIS/MTM-201/14

M.Sc. 2nd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Fluid Mechanics*)

PAPER—MTM-201

Full Marks : 50

Time : 2 hours

Answer **Q. No. 6** and any **three** questions from the rest

The figures in the right-hand margin indicate marks

1. (*a*) A source of fluid situated in space of two dimensions is of such strength that $2\pi\rho\mu$ represents the mass of fluid of density ρ emitted per unit of time. Show that the force necessary to hold a circular disc at rest in the plane of source is

$$\frac{2\pi\rho\mu^2 a^2}{r(r^2 - a^2)}$$

(*Turn Over*)

where a is the radius of the disc and r the distance of the source from its centre. 5

(*b*) Show that the complex potential

$$W = U(z + a^2/z) + ik \log z$$

represents a possible flow past a circular cylinder, sketch the stream lines. Find the stagnation points and calculate the force on the cylinder. 7

2. (*a*) Determine the velocity potential and stream function when an elliptic cylinder is rotating with constant angular velocity w in an infinite mass of the liquid at rest at infinity. 6

(*b*) Show that with proper choice of units the motion of an infinite liquid produced by the motion of an elliptic cylinder parallel to one of its principal axes is given by the complex function

$w = e^{-\zeta}$ where $z = 2 \cosh\zeta$, $(z = x + iy,$
$\zeta = \xi + i\eta)$. 6

3. (*a*) A stream of water of great depth is flowing with uniform velocity V over a plane level bottom. A hemisphere of weight W in water and of radius a rests with its base on the bottom. Prove that the average pressure between the base of the hemisphere and the bottom is less that the fluid pressure at any point of the bottom at a great distance from the hemisphere if

$$V^2 > \frac{32W}{11\pi a^2 \rho}.$$ 6

(*b*) Two vortex filaments of strengths k_1 and k_2 of same sign situated at A_1 and A_2 moves with velocities (u_1, v_1) and (u_2, v_2) respectively. Show that the point G, the centroid of k_1 and k_2 is at rest. Also show that the line A_1A_2 rotates about G with constant angular velocity

$$\frac{k_1 + k_2}{2\pi (A_1A_2)^2}.$$ 6

4. (*a*) An infinite cylinder of radius a and density ρ is surrounded by a fixed cylinder of radius

b, and the intervening space is filled with liquid of density ρ. Prove that the impulse per unit length necessary to start the inner cylinder with velocity V is

$$\frac{\pi a^2}{b^2 - a^2}\left[(\sigma+\rho)b^2-(\sigma-\rho)a^2\right]V.$$ 5

(*b*) Assuming the necessary stress-strain rate relations, deduce the Navier-Stokes equations of motion for a viscous compressible fluid with constant viscosity. 7

5. (*a*) Discuss Prandtl's theory of the boundary layer and obtain the differential equation of motion in the layer along a plate. 6

(*b*) Find the velocity distribution in an incompressible viscous fluid of infinite expanse adjacent to an infinite flat plate which is impulsively started from rest at time $t=0$ and then moves in its own plane with a constant velocity U. Find the thickness of the boundary layer at time t. 6

6. Answer any *two* questions : 2 × 2

(*i*) State Blasius theorem

(*ii*) Define Vortex line and Vortex filament

(*iii*) Reynolds number.

[*Internal Assessment* : 10 Marks]