M.Sc. 1st Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Classical Mechanics and Non-linear Dynamics*)

PAPER – MTM- 105

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **four** questions from the rest

The figures in the right hand margin indicate marks

1. Answer any *four* questions : 2 × 4

   (*a*) What do you mean by generalised forces ? Find the expression of it in terms of generalised coordinates.

   (*b*) Show that the rate of change of angular momentum is equal to the applied torque for a system of particles.

(Turn Over)

(*c*) State Liouville's theorem.

(*d*) Give an example of a mechanical system where

(*i*) Work done by the forces of constraint is zero

(*ii*) Work done by the forces of constraint is non-zero.

(*e*) Describe the phase portrait in the xy-plane of a system described by the following differential equations :

$$\frac{dx}{dt} = 0, \ \frac{dy}{dt} = 0.$$

(*f*) State the fundamental postulates of special Theory of relativity.

2. Derive Lorentz transformation equations in special theory of relativity. 8

3. Discuss a method to determine the eigen frequencies and normal modes of small oscillation of a dynamical system. 8

4. Show that there is no analytic solution for the Euler's equations

$$A\dot{w}_1 - (B-C)w_2 w_3 = 0$$
$$B\dot{w}_2 - (C-A)w_1 w_3 = 0$$
$$C\dot{w}_3 - (A-B)w_1 w_2 = 0$$

where A, B, C are moments of inertia and w_1, w_2, w_3 are the components of angular velocity. 8

5. (*a*) Deduce Hamilton's equations of motion from Hamilton's principle. 4

(*b*) The potential energy and kinetic energy of a dynamical system are given by

$V = \frac{1}{2} kr^2$ and $T = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} mr^2 \sin^2\phi \, \dot{\varphi}^2$.

Determine the Lagrangian and Lagrange's equations of motion. 4

6. (*a*) Prove that

$$\frac{dH}{dt} = -\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t}$$

where H is the Hamiltonian and L is the Lagrangian of a mechanical system composed N particles with n degrees of freedom. 4

(*b*) If the kinetic energy of a conservative scleronomic system is a homogeneous quadratic function of velocities, then prove that the total energy of the system is a constant of motion. 4

7. (*a*) Prove that the Poisson bracket of two constants of motion is itself a constant even, when the constants depend on time explicitly. 4

(*b*) If the transformation equations between two sets of coordinates are

$$P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p,$$

$$Q = \log_e (1 + \sqrt{q}\cos p),$$

Then show that

(*i*) the transformation is canonical, and

(*ii*) the generating function of this transformation can be put in the form

$$F = -(e^Q - 1)^2 \tan p.$$ 4

[*Internal Assessment* — 10 *Marks*]