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M.Sc. 4th Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Non-linear Optimization/Dynamical Oceanology* - II)

PAPER—MTM-404

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks.

MTM-404 (OR)

(*Non-linear Optimization*)

Q. No. 1 is compulsory and answer any **three** from the rest

1. Answer any *five* from the following : 2 × 5

(*a*) What are the basic difference between first existence and second existence theorems in connection with non-linear equations ?

(*Turn Over*)

(*b*) What do you mean by theorems of the alternative ?

(*c*) What do you mean by differentiable convex functions ?

(*d*) Define a continuous game. For a continuous game define the terms mixed strategy, optimal strategy and value of the game.

(*e*) Derive the importance of geometric programming in non-linear programming procedure.

(*f*) For an non-linear programming problem write down the sufficient Kuhn-Tucker conditions.

(*g*) Define maximin and minimax principles with example in a continuous game.

(*h*) Define convex programming problem.

2. (*a*) State the Fritz-John saddle point necessary optimization theorem. 2

(*b*) Use geometric programming to

Minimize $f(x) = 16x_1x_2x_3 + 4x_1x_2^{-1} + 2x_2x_3^{-2} + 8x_1^{-3}x_2$

$x_j \geq 0,\ j = 1, 2, 3$ 6

(*c*) Let X^o be an open set in R^n, let θ and g be defined on X^o. Find the conditions under which a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely. 2

3. (*a*) Let X be an open set in R^n and θ and g be differentiable and convex on X and let $\bar{x}$ solve the minimization problem and let g satisfy the Kuhn-Tucker constraint qualification. Show that there exist a $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ solves the dual maximization problem and $\theta(\bar{x}) = \psi(\bar{x}, \bar{u})$. 5

(*b*) Find geometric-arithmetic mean inequality for a geometric programming problem. 3

(*c*) State 'Linear certainty-equivalence theorem' for one-stage stochastic problem. 2

4. (*a*) Let θ be a numerical differentiable function on an open convex set $\Gamma C R^n$. Prove that θ is convex on Γ if and only if

$$\theta(x^2)-\theta(x^1) \geq \nabla\theta(x^1)(x^2-x^1)$$

for each $x^1, x^2 \in \Gamma$.

Give the geometrical interpretation of the above result. 4 + 2

(*b*) State and prove Motzkin's theorem of the alternative. 4

5. Derive Wolfe's method to solve a quadratic programming problem and hence solve the quadratic programming problem

Max $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to

$$x_1 + 2x_2 \leq 2$$
$$x_1,\ x_2 \geq 0.$$

4 + 6

6. (*a*) State and prove the Fritz-John stationary point necessary optimality theorem. 5

(*b*) Solve by Kuhn-Tucker method

Maximize $x_1^2 + 4x_1x_2 + x_2^2$

subject to $x_1^2 + x_2^2 = 1$ 5

[*Internal Assessment* : 10 Marks]

MTM-404 (OM)

(*Dynamical Oceanology* - II)

Answer any **four** questions

1. Deduce the equations of motion of thermal wind. Hence deduce Taylor-Proudman theorem. 10

2. Obtain the expression of wave function of Poincare-Kelvin wave. 10

3. Obtain the solution of the equation of motion for the pure drift currents in a finitly deep, plane,

homogeneous layer of fluid which rotates uniformly about a vertical axis. Hence deduce the following :

(*i*) The surface current U_s is directed at an angle 45° to the right of the wind stress vector τ in the northern hemisphere.

(*ii*) At a certain depth, the current vector is opposite to U_s. 6 + 2 + 2

4. Write down the vertical structure equation and hence show that the higher baro-clinic mode will propagate its energy more slowly than the barotropic modes. 10

5. In two-dimensional model of ocean current, solve the problem of viscous boundary layer and show that weak back flow appears close to the external edge of the boundary layer. 10

6. Establish the condition for the existence of inertial boundary layer in a two dimensional model ocean. 10

[*Internal Assessment* : 10 Marks]
