PG/IIIS/MTM-303/14

M.Sc. 3rd Semester Examination, 2014

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Dynamical Oceanology and Meteorology/ Operation Research)

PAPER – MTM-303

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

(Dynamical Oceanology and Meteorology)

Answer **Q.No.1** and any **four from** the rest

1. Answer any *four* questions : 2 × 4

   (*a*) Define Gibb's function.

   (*b*) What are β-plane approximations ?

(*c*) Define salinity of sea water.

(*d*) What do you mean by absolute concentration of sea water ?

(*e*) Explain the term 'Tephigram'.

(*f*) What do you mean by non-viscous stratified fluid ?

2. (*a*) Establish the equation

$$Td\eta = dE + pd\gamma - \mu ds$$

(symbols have their usual meanings). 4

(*b*) What are meant by Boussinesq's approximations ? What is the importance of this method ? 4

3. Obtain the equation of motion of sea water in vector form. 8

4. Derive the condition of stability of equilibrium for a stratified fluid. 8

5. (*a*) Show that the sound velocity in sea-water can be expressed as $c^2 = \frac{1}{\rho k_n}$. 4

(*b*) Show that the differential equation for hydrostatic pressure can be expressed in the form

$$\frac{dp}{p} = Xdx + Ydy + Zdz.$$ 4

6. (*a*) Differentiate between isothermal and isentropic process. 4

(*b*) State and prove the equation of state of moist air. 4

7. Establish the Poisson's equation in the following form

$$\frac{T}{\theta} = \left(\frac{p}{100}\right)^k$$

Deduce the temperature lapse rate Γ_d for dry adiabatic atmosphere. 8

8. What do you mean by entropy of a system? Derive the equation of entropy evaluation for sea-water. 2 + 6

[*Internal Assessment – 10 Marks*]

(*Operation Research*)

Answer **Q.No.1** and any **two** from the rest

1. Answer any *four* questions : 2 × 4

   (*i*) What is the usefulness of post optimal analysis ?

   (*ii*) What is the basic difference between simplex and revised simplex methods ?

   (*iii*) Define Quadratic programming problem with an example.

(*iv*) What do you mean by variational principle for unconstraint problem in control theory.

(*v*) How can you define integer programming problem with the help of linear programming problem ?

2. (*a*) Find the curve $x = x(t)$ which minimizes the functional

$$J = \int_0^1 \left[\left(1 + \frac{d^2x}{dt^2} \right)^2 \right] dt$$

subject to the boundary conditions $x(0) = 0$, $x(1) = 1, \dot{x}(0) = 1, \dot{x}(1) = 1$. 8

(*b*) Solve the following non-linear programming problem by using Kuhn-Tucker method

$$\text{Max. } Z = -x_1^2 + 2x_1 + x_2$$

subject to the constraints

$$2x_1 + 3x_2 \leq 6$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0.$$ 8

3. (*a*) Derive Wolfe's method to solve a quadratic programming problem and use the method to solve the following problem

$$\text{Max. } Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

subject to the constraints

$$x_1 + 2x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

3 + 7

(*b*) Using dynamic programming devide a positive quantity C into *n* parts in such a way that their product is maximum. 6

4. (*a*) Find the optimum solution of the linear programming problem

$$\text{Max. } Z = -x_1 + 2x_2 - x_3$$

subject to the constraints

$$3x_1 + x_2 - x_3 \leq 10$$
$$-x_1 + 4x_2 + x_3 \geq 6$$
$$x_2 + x_3 \leq 4$$
$$x_j \geq 0 \text{ for } j = 1, 2, 3$$

Determine the ranges for discrete changes in the components of requirement vectors so as to maintain the feasibility of the existing optimum solution. 8

(*b*) Solve the following linear programming problem by revised simplex method

Max. $Z = 2x_1 + 3x_2 - x_3 + 4x_4 + x_5 - 3x_6$

subject to the constraints

$$x_1 - 2x_2 + x_4 + 4x_5 + \frac{1}{2}x_6 \leq 10$$

$$x_1 + x_2 + 3x_3 + 2x_4 + x_5 - x_6 \leq 16$$

$$2x_1 + \frac{1}{2}x_2 - x_3 - x_4 + 2x_5 + 5x_6 \leq 8$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.$$ 8

[*Internal Assessment – 10 Marks*]