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**PG/IIIS/MTM-302/14**

# M.Sc. 3rd Semester Examination, 2014

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

( *Transform and Integral Equations*)

PAPER — MTM- 302

*Full Marks* : 50

*Time* : 2 hours

Answer **Q.No.1** and any **three** from the rest

*The figures in the right hand margin indicate marks*

1. Answer any *five* questions of the following : $2 \times 5$

   (*a*) Define the term convolution on Fourier transform.

   (*b*) Define eigen value and eigen function of an Integral equation.

( *Turn Over* )

(*c*) Find the value of $f(0)$ and $f'(0)$, when

$$\bar{f}(p) = \frac{1}{p(p^2 + a^2)}$$

using Initial value theorem for Laplace transform.

(*d*) Prove that the convolution operator for Laplace transform is commulative.

(*e*) Define wavelet function and analyze the parameters involving in it.

(*f*) Define infinite Fourier transform and state the conditions of existence of the transform.

2. (*a*) Let $F(k)$ and $G(k)$ be the Fourier transforms of $f(x)$ and $g(x)$ respectively defined in $(-\infty, \infty)$. Show that the Fourier transform of

$$\int_{-\infty}^{\infty} f(u)g(x-u)du$$

can be expressed in terms of the product

$F(k)\,G(k)$. Hence prove that parseval's relation

$$\int_{-\infty}^{\infty} |F(k)|^2\,dk = \int_{-\infty}^{\infty} |f(x)|^2\,dx.$$ 5

(*b*) Using Laplace transform find the solution of the differential equation.

$$t\frac{d^2y}{dt^2} + \frac{dy}{dt} + ty = 0$$

satisfying the condition $y(0) = 1$ and $y'(0) = 0$. 5

3. (*a*) Reduce the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda xy = 1, \ 0 \le x \le l$$

with boundary conditions $y(0) = 0, y(e) = 1$ to an integral equation and find its Kernel. 5

(*b*) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform. 5

4. (*a*) Solve the integral equation

$$y(x) = f(x) + \lambda\int_{-1}^{1}(xt + x^2t^2)y(t)dt. \qquad 5$$

(*b*) State initial value theorem in respect of Laplace transform. Evaluate

$$L\left\{\int_0^t \frac{\sin u}{u} du\right\}$$

by the help of initial value theorem. 1+ 4

5. (*a*) If the Fourier sine transform of $f(x)$ is $\frac{\alpha}{1+\alpha^2}$, $\alpha$ being the transform parameter, then find $f(x)$. 3

(*b*) Find the value of $\sin(t) * t^2$ where $*$ denotes the convolution operator on Laplace transform. 3

(*c*) Under certain conditions (to be specified by you), convert the following integral equation $f(x) = \int_0^x k(x,t)y(t)dt$ into the Volterra integral equation of 2nd kind and then solve it. 4

6. (*a*) Explain a method for solving the integral equation

$$\varphi(x) = f(x) + \lambda \int_a^b \left\{ \sum_i \alpha_i(x)\beta_i(x) \right\} \phi(t)\,dt$$

where the functions $f$, $\alpha_i$, $\beta_i$ are integrable on $[a, b]$ and $\lambda$ is non-zero constant. 6

(*b*) Show that the Fourier transform of

$$\frac{a}{x^2 + a^2},\ (a > 0),\ \text{is}\ \sqrt{\frac{2}{x}}\ e^{-a|k|}.$$

where $k$ is the Fourier transform parameter. 4

[*Internal Assessment* — 10 *Marks*]