# M.Sc. 3rd Semester Examination, 2014

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

*( Partial Differential Equations and Generalized Functions )*

PAPER—MTM-301

*Full Marks* : 50

*Time* : 2 hours

*The figures in the right-hand margin indicate marks*

*( Partial Differential Equations )*

[ *Marks* : 50 ]

Answer **Q. No. 1** and any **two** questions from the rest

1. Answer any *two* questions : 4×2

(*a*) Define Dirac delta function. Let $f(t)$ be any continuous function. Then show that

$$\int_{-\infty}^{\infty} \delta(t-a) f(t) dt = f(a)$$

where $\delta(t)$ is the Dirac delta function.

(*b*) What is the adjoint of a second order linear partial differential operatior ? Find the adjoint of the differential operator $L(u) = u_{xt} - u_x$.

(*c*) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$, where $c$ is constant.

**2.** (*a*) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + x \frac{\partial^2 z}{\partial y^2} = 0, \ (x > 0),$$

to canonical form. 8

(*b*) State Basic existence theorem for Cauchy problem. 3

(*c*) Show that Cauchy problem for the non-homogeneous wave equation is well-posed in the domain $-\infty < x < \infty$, $0 < t \leq T$ where $T > 0$ is fixed. 5

3. (*a*) Solve interior Neumann problem for Laplace equation for a circular disc of radius $a$. 7

(*b*) Let $u$ be a harmonic function on the whole plane such that $u = (3 \sin 2\theta + 1)$ on the circle $x^2 + y^2 = 2$. Without finding the exact form of the solution, find the value of $u$ at the origin. 3

(*c*) State weak maximum principle and strong maximum principle. What is the difference between these two principles? Hence deduce that the Dirichlet Problem

on $D$ ($D \subseteq \mathbb{R}^2$ and bounded) –

$\Delta u = f$ on $D$,

$u = g$ on $\partial D$ where $g$ is a continuous function on $\partial D$, has at most one solution. 2 + 2 + 2

4. (*a*) Define the following with examples-Test function, Distribution. 2 + 2

(*b*) Define the derivative of the Dirac delta function $\delta$ in terms of the derivative of a test function. Show that in the sense of generalised function $H'(x) = \delta(x)$, where $H(x)$ is the Heaviside function and $\delta(x)$ is the Dirac delta function. 2 + 4

(*c*) Consider the Second-order differential equation

$$\frac{d^2u}{dx^2} = f(x)$$

with boundary conditions $u(0) = u(1) = 0$. Find the solution of the problem using Green's function. 6

[ *Internal Assessment* : 10 Marks ]