M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM- 201

(*Fluid Dynamics*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 6** and any **three** questions from rest

The figures in the right hand margin indicate marks

1. (*a*) In the case of two-dimensional motion of a liquid past a fixed circular disc, the velocity at infinity is u in a fixed direction is u, where u is variable. Show that the maximum value of the velocity at any point of the liquid is $2\,u$. Prove also that the force necessary to hold the disc is $2m\dot{u}$, where m is the mass of the liquid displaced by the disc.

(*b*) Show that the complex potential for a liquid past a fixed elliptic cylinder with velocity U parallel to the major axis of the section is given by

$$w = U(a+b)\cosh(\rho - \alpha),$$

where the symbols have their usual meaning. 6 + 6

2. (*a*) In the two dimensional irrotational motion of a liquid streaming part a fixed elliptic disc $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the velocity at infinity being parallel to the major axis and equal to V, prove that if

$$x + iy = c\cosh(\xi + i\eta),\ a^2 - b^2 = c^2$$

$a = c\cosh\alpha$, $b = c\sinh\alpha$, the velocity at any point is

$$\vec{q}^{\,2} = V^2\left(\frac{a+b}{a-b}\right)\frac{\sinh^2(\xi-\alpha)+\sin^2\eta}{\sinh^2\xi+\sin^2\eta}$$

and that it has its maximum value $\frac{V(a+b)}{a}$ at the end of the minor axis.

(*b*) An infinite ocean of an incompressible liquid of density ρ is streaming past a fixed spherical obstacle of radius 'a'. The velocity is uniform and equal to U except in so far as it is disturbed by the sphere, and the pressure in the liquid at a great distance from the obstacle is Π. Show that the thrust on that half of the sphere on which the liquid impinges is 7 + 5

$$\pi a^2\left(\pi - \frac{1}{16}\rho U^2\right).$$

3. (*a*) Consider two parallel rows of vortices one below the other such that the upper vortices are located at $(0, 0)$, $(\pm a, 0)$, $(\pm 2a, 0)$ .... and the lower vortices are located at $(0, -b)$, $(\pm a, -b)$, $(\pm 2a, -b)$, ... If each vortex in the upper row has strength k and each vortex in the lower row is of strength $-k$, show that the vortex system moves with uniform velocity

$$\frac{k}{2a}\coth\frac{\pi b}{a}.$$

(*b*) The circle $|z+a| = a$ is placed in an on coming wind of velocity U and there is a circulation k. Find the complex potential and show that the moment about the origin is $\rho k a U$, where e being the density of the fluid. 6 + 6

4. (*a*) Assuming the necessary stress-strain rate relation, deduce Navier-Stokes equations of motion (in cartesian coordinates) for the incompressible viscous fluid.

(*b*) Find the velocity distribution in an incompressible viscous fluid of infinite expanse adjacent to an infinite flat plate which is impulsively started from rest at time $t = 0$ and then moves in its own plane with a constant velocity U. Find the thickness of the boundary layer at time t. 6 + 6

5. (*a*) Determine the velocity distribution in the steady flow of uniform incompressible viscous fluid between two coaxial circular pipe under the action of a uniform pressure gradient along the common axis of the pipes.

(*b*) Deduce Prandtl boundary layer equations in two dimensional flow of a viscous liquid and the corresponding boundary conditions. 6 + 6

6. Answer any *one* question : 4 × 1

(*a*) A circular cylinder of radius 'a' is fixed across a stream of velocity 'U' with a circulation k round the cylinder. Show that the maximum velocity in the liquid is $2U + \frac{k}{2\pi a}$.

(*b*) Consider the viscous in compressible flow between parallel planes when one plate is fixed and the other is moving with uniform velocity u_0 in its own plane. Find velocity.

[*Internal Assessment* : 10 Marks]
