M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Classical Mechanics*)

PAPER – MTM-105

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **two** questions from the rest

The figures in the right hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) What do you mean by a connected holonomic system ? Give an example of constraints for such a system.

(*b*) Derive the following differential operator

$$\left(\frac{d}{dt}\right)_{fix} \equiv \left(\frac{d}{dt}\right)_{rot} + wx,$$

the symbols having usual meaning.

(*c*) Define Lagrangian and Hamiltonian of a dynamical system.

(*d*) State Hamilton's principle.

(*e*) How the Canonical transformation be determined from the generating function? Explain.

(*f*) State basic postulates of special theory of relativity.

2. (*a*) Show that the path followed by a particle in sliding from one point to another in the absence of friction in the shortest time is a cycloid. 8

(*b*) Derive the Hamilton's equations for conservative unconnected holonomic system. 8

3. (*a*) A body moves under no forces about a point 0, the principal moments of inertia at 0 being $6A$, $3A$ and A. Initially, angular velocity of the body has the components $w_1 = n$, $w_2 = 0$, $w_3 = 3n$ about the principal axes. Show that at any time t, $w_2 = -\sqrt{5}n \tan h \sqrt{5}nt$ and ultimately the body rotates about the mean axis. 8

(*Continued*)

(*b*) If a body in the northern hemisphere falls freely to the ground from a height h, show that it trikes the ground at

$$\frac{2}{3}\omega h\left(\frac{2h}{g_e}\right)^{1/2}\cos\lambda$$

to the east, where ω is the earth's angular velocity, g_e is the acceleration due to the combined effect of gravity and centrifugal force and λ is the latitude of the place. 8

4. (*a*) Derive the Lorentz transformation equations for relativistic mechanics. 8

(*b*) Prove that the Poisson bracket of two constants of motion is itself a constant even when the constants depend on time explicitly. 4

(*c*) Use Hamilton's procedure, to find the equation of motion of a simple pendulum. 4

[*Internal Assessment* : 10 Marks]