M.Sc. 3rd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Operations Research / Dynamical Oceanology and Meterology*)

PAPER—MTM-303

The figures in the right-hand margin indicate marks.

(*Operations Research*)

[*Marks* : 50]

Time : 2 hours

Answer **Q. No. 1** and any *two* from the rest

1. Answer any *four* questions : 2 × 4

(*a*) If in the EOQ model with uniform demand, no shortages and infinite rate of replenishment, the purchasing cost, p per unit quantity is considered, then show that there will be no change in the optimum order quantity.

(*b*) What is "transversality condition" ? Explain it.

(*c*) Write down the type of the LPP suited for the application of Decomposition Principle. Mention at least its one advantage.

(*d*) Explain 'Stage' and 'State' in Dynamic Programming Method.

(*e*) What do you mean by Integer programming problem ?

(*f*) What are the basic difference between simplex method and revised simplex method ?

2. (*a*) Find the stationary path $x = x(t)$ for the functional

$$J = \int_0^1 \left[1 + \left(\frac{d^2x}{dt^2}\right)^2\right] dt$$

subject to the boundary conditions $x(0) = 0$, $x(1) = 1$, $\dot{x}(0) = 1$, $\dot{x}(1) = 1$. 8

(*b*) Reduced the following LPP in to a more convenient form of LPP using revised simplex method. 8

Minimize $Z = -5x_1 - 3x_2 - 5x_3 - 2{\cdot}5x_4$

subject to

$$2x_1 + 3x_2 + 3x_3 + 2x_4 \le 15$$
$$x_1 + x_2 \le 4$$
$$2x_1 + x_2 \le 6$$
$$2x_3 + 5x_4 \le 10$$
$$x_3 \le 4$$
$$x_1,\ x_2,\ x_3,\ x_4 \ge 0$$

3. (*a*) Using dynamic programming, find

$$\text{Min} \quad Z = x_1 + x_2 + x_3 + \ldots.. + x_n$$

where $x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n = d$

$x_1, x_2, \ldots x_n > 0.$ 8

(*b*) Find the optimum order quantity for an inventory control system with finite rate of production, no shortages, uniform demand, zero leadtime and other usual assumptions. Find optimum time period also. Deduce the EOQ formula from it. 8

4. (*a*) Using Beale's method, solve the following quadratic programming problem **8**

Max $Z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

subject to $x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0.$

(*b*) Using method of constrained variation **8**

Minimize $f(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + x_3^2$

subject to $x_1 + x_2 + x_3 = 5$

$2x_1 + 3x_2 + x_3 = 9$

5. (*a*) Using revised simplex method **8**

Maximize $Z = x_1 + x_2$

subject to $3x_1 + 4x_2 = 7$

$4x_1 + 3x_2 = 7$

$x_1, x_2 \geq 0.$

(*b*) Consider the LPP **8**

Maximize $Z = x_1 + x_2$

subject to $2x_1 + x_2 \leq 6$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

Find the optimal solution. Using this optimal table find the optimal solution when the objective function is $z = 3x_1 + x_2$.

[*Internal Assessment* : 10 Marks]

(*Dynamical Oceanology and Meterology*)

[*Marks* : 50]

Time : 2 hours

Answer any *five* questions

The symbols have their usual meanings

1. Define Gibb's function. Establish Gibb-Duhem relation for two component sea-water. 2 + 6

2. What do you mean by the entropy of a system ? Derive the equation of entropy evaluation for sea-water. 2 + 6

3. State clearly the assumptions of Boussineoq's approximation. What are β-plane approximations ? Write down the expressions for the heat flux density and the diffusive salt flux density for sea-water. 3 + 2 + 3

4. (*i*) Show that the velosity of sound in sea-water can be expressed as 4

$$C^2 = \frac{1}{\rho K_\eta}.$$

(*ii*) Show that the differential equation for hydrostatic pressure can be written as 4

$$\frac{dp}{\rho} = Xdx + Ydy + Zdz$$

5. Establish the Poisson's equation in the following form

$$\frac{T}{\theta} = \left(\frac{p}{1000}\right)^K.$$

Deduce the temperature lapse rate Γ_d for dry adiabatic atmosphere. 5 + 3

6. Show that the equation of state of moist air can be expressed as 8

$$\frac{1}{\bar{m}} = \frac{1}{m_d} \cdot \left(\frac{1 + \frac{\omega}{\varepsilon}}{1 + \omega} \right).$$

7. Explain the terms 'Tephigram' and 'Emagram'. Discuss cyclogenesis. 3 + 5

[*Internal Assessment* : 10 Marks]