M.Sc. 3rd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-302

Full Marks : 50

Time : 2 hours

Answer **Q.No.1** and any **three** from the rest

The figures in the right hand margin indicate marks

(*Integral Transforms and Integral Equations*)

1. Answer any *five* questions : 5 × 2

(*a*) Define singular integral equation with an example.

(*b*) Define exponential order regarding Laplace transform with an example.

(*c*) What do you mean by inverse Fourier transform of a special function ?

(*d*) Define eigen value and eigen function of an integral equation.

(*e*) Give an example to show that the integral of a good function is not necessarily a good function.

(*f*) Find the Mellin transform of $\frac{1}{(1+x)^n}$.

2. (*a*) Discuss the solution procedure of homogeneous Fredholm integral equation of the second kind with degenerate kernel. 5

(*b*) Find the exponential Fourier transform of $f(t)$ where 5

$$f(t)=\begin{cases}1-|t|, & |t|<1\\ 0, & |t|>1\end{cases}$$

3. (*a*) Solve the following problem of conduction of heat in an infinite circular cylinder

$$\frac{\partial u}{\partial t}=\lambda\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right),\ 0\le r\le a,$$

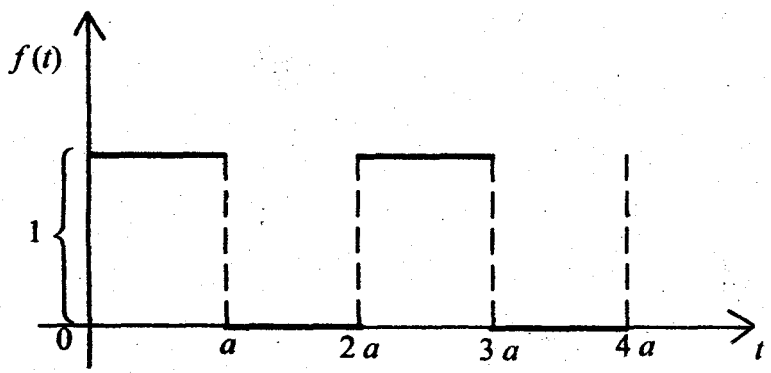
subject to $u(a,t)=0, u(r,0)=f(r), 0\le r\le a$. 5

(*b*) Solve

$$y''(t) + t\,y'(t) - 2y(t) = 4, \quad y(0) = -1, y'(0) = 0,$$

by Laplace transform technique. 5

4. (*a*) Find the Laplace transform of the periodic function $f(t)$, whose graph is given below : 4



(*b*) Find the resolvent kernel of the following integral equation and hence solve it

$$y(x) = e^x + \lambda \int_0^{10} xt\; y(t)dt.$$ 6

5. (*a*) Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} - (\sin x)\frac{dy}{dx} + e^x y = x$$

with the initial conditions $y(0) = 1$, $y'(0) = -1$. 6

(*b*) If the Fourier cosine transform of $f(x)$ is $\frac{1}{a\sqrt{2}} e^{-a^2/4a}$, then find $f(x)$. 4

[*Internal Assessment* – 10 Marks]