M.Sc. 2nd Semester Examination, 2012

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Stochastic Process and Regression*)

PAPER—MTM-206

Full Marks : 25

Time : 1 hour

Q. No. 1 is compulsory and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* from the following : 2 × 2

(*a*) Define the terms : Ergodic, Irreducible chain. 2

(*b*) What do you mean by partial correlation and multiple correlation coefficient of three variables. 2

(*c*) Suppose P is a stochastic matrix, then show that P^n is also a stochastic matrix for all $n > 1$. 2

2. Describe a linear birth and death process and derive the differential equations describing the process and hence solve it. 8

3. (*a*) Show that if $X_3 = aX_1 + bX_2$, the three partial correlations are numerically equal to unity, $r_{13\cdot2}$ having the sign of a, $r_{23\cdot1}$, the sign of b and $r_{12\cdot3}$, the opposite sign of a/b. 3

(*b*) Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $\{X_n, n \geq 1\}$ be the digit leaving the nth stage of the system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage there is a constant probability q that the digit which enters will be transmitted unchanged (i.e, the digit will remain unchanged when it leaves) and probability p otherwise (i.e., the digit changes when it leaves), $p + q = 1$. Find the one step transition matrix A and n-steps transition matrix A^n. Also find A^n when $n \to \infty$. 5

4. (*a*) State and prove Chapman-Kolmogorov equation. 4

(*b*) When do we say that a state *j* is accessible from a state *i* ? When do we say that the two states *i* and *j* communicate ? 3

(*c*) What do you mean by periodicity of a state ? 1

[*Internal Assessment* : 5 Marks]