M.Sc 4th Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER — MA - 2204

(*Nonlinear Optimization*)

Full Marks : 50

Time : 2 hours

Q.No.1 is compulsory and answer any **three** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* of the following : 2 x 5

(*a*) Define degree of difficulty to solve a geometric programming problem with an example.

(*b*) Under what conditions, the Kuhn-Tucker condition for quadratic programming problem will be necessary and sufficient ?

(*c*) When two bimatrix games are said to be strategically equivalent ?

(*d*) What do you mean by differentiable convex function ?

(*e*) What are basic differences between Wolfe's method and Beale's method ?

(*f*) What do you mean by convex programming problem ? Which type of convex programming problem can be solved by Frank-Wolfe algorithm ?

2. (*a*) Define :

(*i*) The (primal) quadratic minimization problem,

(*ii*) The quadratic dual (maximization) problem.

(*b*) How do you solve the following geometric programming problem ?
Find

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

that minimizes the objective function

$$f(x) = \sum_{j=1}^{N} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}} \right)$$

3 + 7

3. (*a*) Use the chance constained programming technique to find an equivalent deterministic LP problem to the following stochastic programming problem :

$$\text{Minimize} \quad F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \quad P\left[\sum_{j=1}^{n} a_{ij} x_j \leq b_i\right] \geq p_i$$

$$x_j \geq 0, \; i = 1, 2, \ldots, n$$
$$j = 1, 2, \ldots, n$$

where c_j and a_{ij} are random variables and p_i are specified probabilities.

(*b*) Define the following :

(*i*) Fritz-John stationary point problem.

(*ii*) Kuhn-Tucker stationary point problem. 7 + 3

4. (*a*) Solve the following quadratic programming problem by Wolfe's modified simplex method and test wheather the solution is global optimum or not

$$\text{Minimize } f(x) = -8x_1 - 16x_2 + x_1^2 + 4x_2^2$$

$$\text{subject to} \quad x_1 + x_2 \leq 5$$
$$x_1 \leq 3,$$
$$x_1 \geq 0, \; x_2 \geq 0.$$

(*b*) Let $\bar{x} \in X^0$, let X^0 be an open, and let θ and g be differentiate and convex at $\bar{x}$. If $(\bar{x}, \bar{u})$ is a solution of KTP, then prove that $\bar{x}$ is a solution of MP. If $(\bar{x}, r_0, \bar{r})$ is a solution of FJP, and $r_0 > 0$, then prove that $\bar{x}$ is a solution of MP.

7 + 3

5. (*a*) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz-John saddle point problem?

(*b*) Find the Nash equilibrium solution (s) of the following bimatrix game (if exists)

$$\begin{bmatrix} (-2, -1) & (1, 1) \\ (-1, 2) & (-1, -2) \end{bmatrix}$$

7 + 3

6. (*a*) Solve the following quadratic programming problems by using Beale's method :

$$\text{Maximize } Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$$

subject to constraints

$$x_1 + 2x_2 \leq 10$$
$$x_1 + x_2 \leq 9$$
$$x_1, \ x_2 \geq 0.$$

(*b*) Write short note on complementary slackness principle. 7 + 3

[*Internal Assessment* — 10 Marks]

(*Dynamical Oceanology* -II)

Full Marks : 50

Time : 2 hours

Answer any **four** questions

The questions are of equal value

1. Define absolute vorticity. Deduce the general vorticity equation of the fluid.

2. Define Rossby Number. Deduce the equations of thermal wind.

3. Show that the group velocity vector of plane Rossby waves makes an 2α with the axis of x and has a magnitude $\frac{\beta}{K^2}$ (symbols have their usual meaning).

4. Prove that the equation of Ekman layer on a sloping surface can be expressed as

$$\frac{1}{4}\cdot\frac{\partial^4 v}{\partial \zeta^4}+v=\frac{1}{\cos\theta}\frac{\partial p}{\partial x^1},$$

(symbols have their usual meaning).

5. Obtain the solution of the equation of motion for the pure wind drift current in plane homogeneous layer of fluid as the infinite depth rotating uniformly about a vertical axis.

6. Give a mathematical formulation of a linear model of thermocline. Obtain the expression of a perturbation temperature T_s outside the western boundary layer in the following form :

$$T_s=\frac{2\theta(y)}{\pi}\int_0^\infty \frac{\sin \tau x}{\tau}\left(1-e^{x^2 f^2 \tau^2}\right) d\tau$$

where symbols have their usual meaning.

[*Internal Assessment* — 10 Marks]