M.Sc 4th Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-2203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

GROUP – A

(*Fuzzy Sets and their Applications*)

[*Marks* : 25]

Answer **Q. No. 1** and any *three* from the rest

1. Answer any *two* questions : 1 × 2

(*a*) Define the normal fuzzy set.

(*b*) What do you mean by α-cut of a fuzzy set ?

(*c*) Give an example of a triangular fuzzy number.

2. Prove that for fuzzy sets distributive law and De Morgan law are true. 6

3. (*a*) Prove that the law of contraction and law of excluded middle do not hold for fuzzy sets. 4

(*b*) Let $\tilde{A}$ be a fuzzy set in X with the membership function $\mu_{\tilde{A}}(x)$. Let A_α be the α-cuts of $\tilde{A}$ and $\chi_{\tilde{A}}(x)$ be the characteristic function of the crisp set A_α for $\alpha \in (0, 1]$. Then for each $x \in X$, show that

$$\mu_{\tilde{A}}(x) = \sup\{\alpha \wedge \chi_{\tilde{A}}(x) : 0 < \alpha \leq 1\}.$$ 2

4. (*a*) If $\tilde{A} = [a_1, b_1, c_1]$ and $\tilde{B} = [a_2, b_2, c_2]$, then prove that 4

$$\tilde{A} - \tilde{B} = [a_1 - c_2,\ b_1 - b_2, c_1 - a_2].$$

(*b*) State the Zadeh's extension principle. 2

5. Using Werner's method form a crisp LPP corresponding to the following fuzzy LPP : 6

$$\text{Maximize} \quad z = 4x_1 + 5x_2 + 9x_3 + 11x_4$$

$$\text{subject to} \quad x_1 + x_2 + x_3 + x_4 \leq \widetilde{15}$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 \leq \widetilde{80}$$

$$3x_1 + 5x_2 + 10x_3 + 15x_4 \leq \widetilde{100}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

and the tolerances as $p_1 = 5,\ p_2 = 40,\ p_3 = 30.$

6. Using Verdegay's approach, derive the equivalent crisp LPP for the following fuzzy LPP: 6

$$\text{Maximize } Z = CX$$

$$\text{subject to} \quad (AX)_i \lesssim b_i, \ i = 1, 2, \ldots, m$$

$$X \geq 0.$$

[*Internal Assessment* : 5 Marks]

GROUP – B

(*Soft Computing*)

[*Marks* : 25]

1. Answer any *two* of the following : 8 × 2

(*a*) Maximize $f(x) = \sqrt{x}$ in $1 \leq x \leq 16$ using binary coded *GA* (one iteration only) Given that

Population size = $N = 6$

Initial Population, $x(i) \equiv$ 100101
011010
010110
111010
101100
001101

Random Nos. to be used for selection :
·15, ·27, ·64, ·52, ·79, ·70
$p_c = 0{\cdot}70$
Random Nos. for cross-over
0·62, ·80, ·50, ·47, ·75, ·45
$p_m = 0{\cdot}03$

Random Nos. for mutation

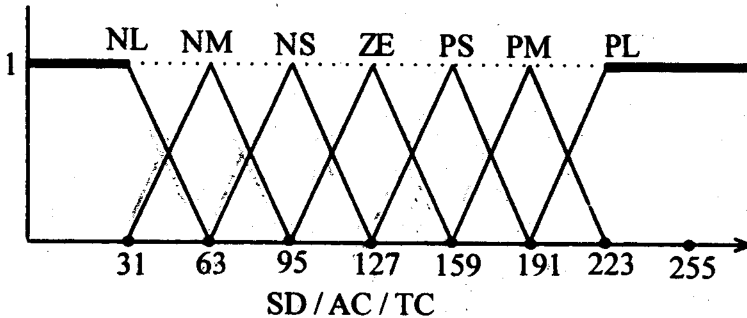
·61, ·21, ·75, ·08, ·04, ·91, ·45, ·11,
·06, ·81, ·05, ·09, ·12, ·41, ·51, ·62,
·78, ·84, ·44, ·90, ·78, ·32, ·07, ·06,
·55, ·15, ·29, ·37, ·77, ·67, ·08, ·02,
·61, ·82, ·92, ·83.

(*b*) A controller is used to maintain a vehicle at a desired speed. The system consists of two fuzzy inputs – speed difference (SD) and acceleration (AC) and one fuzzy output, Throttle control (TC). The fuzzy rule base for the system is

IF (SD is NL) AND(AC is ZE) THEN (TC is PL)
IF (SD is ZE) AND (AC is NL)THEN (TC is PL)
IF (SD is NM) AND (AC is ZE)THEN(TC is PM)
IF (SD is NS) AND(AC is PS)THEN (TC is PS)
IF (SD is PS) AND (AC is NS)THEN (TC is NS)
IF (SD is PL) AND (AC is ZE)THEN(TC is NL)
IF (SD is ZE) AND (AC is NS)THEN(TC is PS)
IF (SD is ZE) AND (AC is NM)THEN(TC is PM)

where NL : Negative Large, NS : Negative Small
NM : Negative Medium, ZE : Zero
PS : Positive Small, PM : Positive Medium
PL : Positive Large

Fuzzy sets for SD, AC and TC (in Normalised form) are :



If the normalised speed difference be 100 and the normalised acceleration be 70, then what should be the Throttle control in normalised form ?

(*c*) Let the classification is as follows :

$$\left\{X_1^T = [2, 2],\ d_1 = 0\right\},\ \left\{X_2^T = [1, -2],\ d_2 = 1\right\}$$

$$\left\{X_3^T = [-2, 2],\ d_3 = 0\right\},\ \left\{X_4^T = [-1, 1],\ d_4 = 1\right\}.$$

Solve it with single vector input, two element perceptron network [2 iterations only].

2. Answer any *one* of the following : 4 × 1

(*a*) Evaluate the following fuzzy ponens :

Premise	a is low
Implication	a and b are approximately equal
Conclusion	b is somewhat low

where

$$\mu_A(a) = \begin{cases} 1\cdot0 & \text{if} \quad 0 \le a \le 10 \\ 0\cdot7 & \text{if} \quad 10 < a \le 12 \\ 0\cdot3 & \text{if} \quad 12 < a \le 13 \\ 0\cdot0 & \text{if} \quad 13 < a \end{cases}$$

$\mu_R(a, b)$ where R is a fuzzy relation "approximately equal", is

	$0 \le a \le 10$	$10 < a \le 12$	$12 < a \le 13$	$13 < a$
$b = 9$	1·0	0·8	0·3	0·0
$b = 10$	1·0	0·9	0·7	0·3
$b = 11$	0·8	1·0	0·9	0·7
$b = 12$	0·5	1·0	1·0	0·9

Or

Explain the following:

(*a*) Advantages of using genetic algorithm in optimization.

(*b*) Arithmetic crossover in genetic algorithm and role of mutation in GA.

[*Internal Assessment* : 5 Marks]