M.Sc 2nd Semester Examination, 2011

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Fluid Dynamics*)

PAPER—MTM-201

Full Marks : 50

Time : 2 hours

Answer **Q. No. 6** and any **three** from rest

The figures in the right-hand margin indicate marks

1. (*a*) State and prove Blasius theorem. 6

(*b*) Show that the force per unit length exerted on a circular cylinder, radius a, due to a source of strength m, at a distance r from the axis is

$$\frac{2\pi\rho m^2 a^2}{r\left(r^2 - a^2\right)}.$$ 6

2. (*a*) A stream of water of great depth is flowing with uniform velocity V over a plane level bottom. An infinite cylinder, of which the cross-section is a semi-circle of radius 'a', lies on its flat side with its generating lines making an angle α with the undisturbed stream lines. Prove that the resultant fluid pressure per unit length on the curved surface is

$$2a\,\Pi - \frac{5}{3}\rho a V^2 \sin^2 \alpha$$

where Π is the fluid pressure at a great distance from the cylinder. 6

(*b*) Show that the complex potential

$$W = U(z + a^2/z) + ik \log z$$

represent a possible flow past a circular cylinder, sketch the stream lines, find the stagnation points and calculate the force on the cylinder. 6

3. (*a*) Show that the motion due to a set of line vortices, each of strength k at points $z = \pm na$, $(n = 0, 1, 2, ...)$, is given by the relation

$$w = \frac{ik}{2\pi} \log \sin\left(\frac{\pi z}{a}\right).$$

Hence find the velocity components. 6

(*b*) Find the complex potential and stream funetion in case of an elliptic cylinder rotating in an infinite mass of liquid at rest at infinity. 6

4. (*a*) A circular cylinder is moving in a liquid at rest at infinity. Calculate the force on the cylinder owing to the presence of the liquid. Also, show that the effect of the presence of the liquid is to reduce the thrust in the ratio $(\sigma - \rho) : (\sigma + \rho)$, where σ is the density of the cylinder and ρ is the density of the fluid. 6

(*b*) The space between two concentric spherical shells of radii a and b $(a > b)$ is filled with an incompressible fluid of density ρ and the shells suddenly begin to move with velocities U and V in the same direction. Prove that the resultant impulsive pressure on the inner shell is 6

$$\frac{2\pi\rho b^3}{3(a^3-b^3)}\left[3a^3U-(a^3+2b^3)V\right].$$

5. (*a*) An incompressible viscous fluid flows in a channel formed by two parallel plates in the presence of an oscillating pressure gradient $k\cos\Omega t$ along the axis of the channel, k being a constant, find the velocity distribution in the flow. 6

(*b*) Determine the velocity distribution in the steady flow of uniform incompressible viscous fluid between two coaxial circular pipe under the action of a uniform pressure gradient along the common axis of the pipes. Find the average velocity and the volume flux. 6

6. Answer either (*a*) or (*b*) : 4

(*a*) Establish the Karman's momentum integral equation relating to motion in the boundary layer.

(*b*) State the significance of Reynolds number.

[*Internal Assessment* : 10 Marks]