PG/IS/MA-1102/10

# M.Sc 1st Semester Examination, 2010

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

( *Complex Analysis* )

PAPER—MA-1102

*Full Marks* : 50

*Time* : 2 hours

Answer **Q. No. 4** and any **two** from the rest

*The figures in the right-hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

**1.** (*a*) Obtain a set of sufficient condition for a function to be analytic. 4

(*b*) Determine a function $v(x, y)$ such that the function $f = u + iv$ is analytic on $C$, the complex plane, where $u(x, y) = x(1 - y)$. 4

(*c*) If a function $f(z)$ is continuous on a contour $C$ of length $l$ and if $M$ be the upper bound of $|f(z)|$ on $C$ then 4

$$\left| \int_C f(z)\,dz \right| \leq Ml$$

(*d*) Let $f(z) = u + iv$ be analytic in a region. Prove that

$$\frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2.$$ 4

**2.** (*a*) Evaluate : 4

$$\int_C \frac{z+4}{z^2+2z+5}\,dz$$

where $C$ is the circle $|z+1| = 1$.

(*b*) State and prove Rouche's theorem. 4

(*c*) Prove that the transformation

$$w = \frac{az+b}{cz+d},\ ad - bc \neq 0$$

transforms circles in the $z$-plane into circles in the $w$-plane, straight line using considered as limiting cases of circles. 4

(*d*) Show that all the roots of the equation

$$z^5 - 12z^2 + 14 = 0$$

lie between the circles $|z|=1$ and $|z|=\frac{5}{2}$. 4

3. (*a*) Find Laurent series at $z = 1$ where

$$f(z) = \frac{e^{2z}}{(z-1)^3}.$$

Name the singularity. 4

(*b*) Find the residues of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$$

at all its poles in the finite plane. 4

(*c*) Evaluate the following by the method of contour integration (any *two*): 4 x 2

(*i*) $\int_0^\infty \frac{dx}{(1+x^2)^2}$.

(*ii*) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx.$

(*iii*) $\int_{0}^{\infty} \frac{\sin \pi x}{x(1-x^2)}\,dx.$

(*iv*) $\int_{0}^{2\pi} \frac{d\theta}{5+3\cos\theta}.$

**4.** Answer the following: 2 x 4

(*a*) When is a function $f(z)$ said to be analytic in a given domain in the complex $z$-plane?

(*b*) Prove that:

$$u = e^{-x}\,(x \sin y - y \cos y)$$

is harmonic.

(*c*) Locate and name the singularity of

$$f(z) = \frac{z}{(z^2+4)^2}.$$

(*d*) Find Res $f(z)$ at $z = 0$ where

$$f(z) = \frac{z-3}{z^2} \sin \frac{1}{1-z}.$$

[*Internal Assessment* : 10 Marks ]