M.Sc. 4th Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-2205

The figures in the right-hand margin indicate marks

(*Operational Research Modelling* - II)

Paper — MA 2205 (OR)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and any **two** from the rest

1. Answer any *two* questions : 4

(*a*) What do you mean by information ? Explain with an example.

(*b*) What do you mean by reliability of an item and a system ?

(*c*) Define joint and conditional entropies.

(*d*) Define MTBF when failure distribution is exponential.

2. Find the optimal sequence for processing 4 jobs *A*, *B*, *C*, *D* on 4 machines M_1, M_2, M_3, M_4 in the order $M_1 M_2 M_3 M_4$ with the following processing time: 8

Jobs	Machine (Processing time in hours)			
	M_1	M_2	M_3	M_4
A	15	5	4	14
B	12	2	10	12
C	13	3	6	15
D	16	0	3	19

3. (*a*) Prove that:

$$J = \int_{x_0}^{x_1} F(y_1, y_2, \dots, y_n, y_1', y_2', \dots, y_n', x)\, dx$$

will be stationary only if

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right) - \frac{\partial F}{\partial y_j} = 0 \text{ for } j = 1, 2, \dots, n$$

where $y_j' = \dfrac{\partial y_j}{\partial x}$ 4

(*b*) Derive the differential equations of the lines of propagation of light in an optically non-homogeneous medium with the speed of light $c(x, y, z)$. Also, discuss the case when c is constant. 4

4. (*a*) Define entropy function. The following two finite probability schemes are given by $(p_1, p_2, \dots, p_n)$ and $(q_1, q_2, \dots, q_n)$,

with $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i$. Then show that

$$-\sum_{i=1}^{n} p_i \log p_i \le -\sum_{i=1}^{n} p_i \log q_i$$

with equality iff $p_i = q_i$ for all i. 4

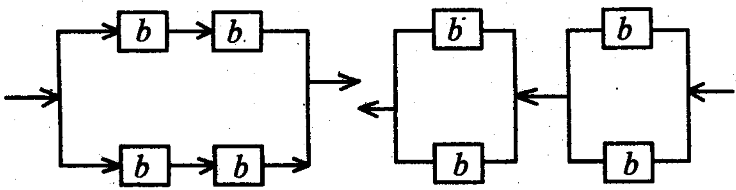
(*b*) Let x_n be a particular event with probability p_n is divided into m mutually exclusive subsets, say $E_1, E_2, \ldots, E_m$ with probabilities $q_1, q_2, \ldots, q_m$ respectively, such that $p_n = q_1 + q_2 + \ldots + q_m$, then prove that

$$\begin{aligned} H(p_1, p_2, \ldots, p_{n-1}, q_1, q_2, \ldots, q_m) &= H(p_1, p_2, \ldots, p_{n-1}, p_n) \\ &\quad + p_n H(q_1/p_n, q_2/p_n, \ldots, q_m/p_n), \end{aligned}$$

where H is the entropy function. 4

5. (*a*) Derive the use of reliability in the design of a system when the system in (*i*) series arrangement, and in (*ii*) parallel arrangement. 4

(*b*) Assume that the reliability of each individual component is b, which design describe below will you prefer and why ? 4



[*Internal Assessment* : 5 Marks]

(*Dynamical Meteorology* - II)

Paper – MA 2205 (OM)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and any **two** from the rest

1. Answer any *one* : 2

(*a*) What is CISK ?

(*b*) Show that in a geostropic wind field, an ideal front is necessarily stationary.

2. Derive the pressure tendency below a frontal surface. What do you mean by frontogenesis and frontolysis? Find the required condition when frontogenesis and frontolysis occur. 5 + 4

3. State the factors that affect the atmospheric air flow near the ground, what is the roughness height? Derive the logarithmic wind law equation. 2 + 1 + 6

4. (*a*) Show that the tangential velocity in a hurricane vary with altitude z by the following relation.

$$\left(\frac{2\,M_{\tan}}{R} + f_c\right)\frac{\Delta\,M_{\tan}}{\Delta z} = \frac{g}{\overline{T}}\,\frac{\Delta T}{\Delta R}$$

where $M_{\tan}$ is the tangential velocity of the hurricane, R is the distance from the eye, T is the absolute temperature, f_c is the Coriolis parameter. Hence show that the hurricane has a warm core. 6

(*b*) In initially rotationless air at radius 400 km from a tropical-cyclone centre, find its tangential velocity after it converges to a radius of 100 km. The cyclone is centered at 10° latitude and the angular velocity of the earth is $0 \cdot 729 \times 10^{-4}\ s^{-1}$. 3

[*Internal Assessment* : 5 Marks]
