M.Sc 4th Semester Examination, 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-2204

(*Nonlinear Optimization*)

Full Marks : 50

Time : 2 hours

Q. No. 1 is compulsory and any **three** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* of the following: 2×5

(*a*) What do you mean by decomposition principle of Dantzig and Wolfe? What is the advantage of decomposition principle?

(*b*) What is the necessity of constraint qualification related with nonlinear programming?

(*c*) What do you mean by polynomial and posynomial?

(*d*) What is chance constrained programming technique?

(*e*) Define bimatrix game with an example.

(*f*) Define convex programming problem with an example.

2. (*a*) Describe Wolfe's modified simplex algorithm to solve the quadratic programming problem.

$$\text{Maximize} \quad Z = f(x_1, x_2, \ldots, x_n)$$

$$= \sum_{j=1}^{n} C_j x_j + \sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk} x_j x_k$$

subject to constraints

$$\sum a_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots, m$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n$$

where $C_{jk} = C_{kj}$, $\forall\, j$ and k, $b_i \geq 0$, $\forall\, i$.

(*b*) State and prove saddle point necessary optimality theorem. 6 + 4

3. (*a*) Let θ be a numerical differentiate function on an open convex set $\Gamma C R^n$. Prove that θ is concave on Γ if and only if

$$\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1),$$

for each x^1, $x^2 \in \Gamma$.

(*b*) When a Nash equilibrium strategy pair is admissible? Prove that all strategically equivalent bimatrix games have the same Nash equilibria. Define Nash equilibrium strategy and Nash equilibrium outcome in mixed strategies. 5 + 5

4. (*a*) Solve by Beale's method

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + 3x_2 - x_1^2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0. \end{aligned}$$

(*b*) State and prove weak duality theorem. 7 + 3

5. (*a*) State and prove Wolfe's duality theorem.

(*b*) Find $x_1 > 0, \; x_2 > 0$ that minimizes

$$g(x_1, x_2) = 8x_1 + \frac{12}{x_1^2 x_2^3} + 3x_2^4.$$ 5 + 5

6. (*a*) State and Fritz John saddle point necessary optimality theorem.

(*b*) State Kuhn - Tucker stationary point necessary optimality theorem.

(*c*) Write the relationship between the solutions of the local minimization problem (LMP), the minimization problem (MP), the Fritz John stationary problem (FJP) and Kuhn - Tucker stationary problem (KTP). 5 + 2 + 3

[*Internal Assessment* : 10 Marks]

(*Dynamical Oceanology* -II)

Full Marks : 50

Time : 2 hours

Answer any **four** questions

The figures in the right-hand margin indicate marks

1. Deduce the momentum equation and state the physical interpretation of the terms. 7 + 3

2. Deduce the equations of inertia-current. Hence find the inertial periods at the pole and equator. 6 + 4

3. Show that the shallow-water equation can be expressed as

$$\frac{dH}{dt} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

(symbols have their usual meaning). 10

4. Discuss Poincaré and Kelvin waves when

$$\alpha = \frac{n\pi}{L}, \ n = 1, 2, 3, \ldots$$ 10

5. Obtain the solution of the equations of motion for the pure drift currents in a finitely deep, plane, homogeneous layer of fluid which rotates uniformly about a vertical axis. Hence deduce the following :

   (*i*) The surface current U_s is directed at an angle 45° to the right of the wind stress vector τ in the northern hemisphere.

   (*ii*) At a certain depth, the current vector is opposite to U_s. 10

6. Write down the vertical structure equation and hence show that the higher baroclinic mode will propagate its energy more slowly than the barotropic modes. 10

[*Internal Assessment* : 10 Marks]

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