M.Sc 3rd Semester Examination 2010

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA - 2106

(Operational Research Modelling - I/ *Dynamical Meteorology* - I)

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(Operational Research Modelling - I)

Answer **Q.No.1** any **four** from the rest

1. Answer any *four* questions : 2 x 4

   (*a*) Derive the minimum average total cost for an EOQ model stating the assumptions.

   (*b*) Explain 'Steady State' and 'Transient State' in a queueing system. Explain when traffic intensity (ϱ) is less than 1 and when this restriction is not required.

(*c*) Explain the 'Group Replacement' policy. Define the 'Mortality Theorem' with the expression of average number of death in steady state.

(*d*) Construct a net work for each of the activities and their precedence relationships are given below.

Activity	Precedence
A	—
B	—
C	*A*
D	*A*
E	*I, J, K*
F	*B, D*
G	*B, D*
H	*F*
I	*A*
J	*G, H*
K	*F*

(*e*) Derive the expression of Total expected cost only for a news-boy problem with discrete units for replenishment and demand against a fixed time period (state all the assumptions — optimum order quantity is not required).

(*f*) Explain the Poisson's axioms in a Poisson queueing system. What do you mean by (*a* / *b* / *c*: *d* / *e*) for a queueing system.

2. Determine the optimum order quantities for three inventory items with the following data, No stock-out is allowed.

Item	1	2	3
Demand (Units per day)	2	4	4
Set-up cost (Rs.)	10	5	15
Holding cost (Rs.)	0·3	0·1	0·2
Floor space required (ft^2/unit)	1	1	1

Total available storage area = 25 ft^2. 8

3. The following mortality rates have been observed for a certain type of light bulbs.

Week :	1	2	3	4	5
Percent failing by the end of week :	10	25	50	60	100

There are 1000 bulbs in use and it costs Rs. 1 to relace an individual bulb which burnt out. If all the bulbs are replaced simultaneously it would

cost 25 paise per bulb. Find the optimum time period for replacement following the group replacement policy. Compare it with the individual replacement policy. 8

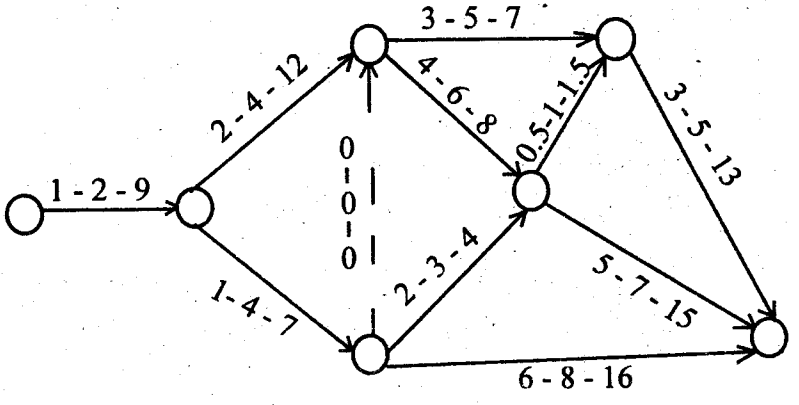
4. (*i*) Define 'dummy activity'. Identify the dummy activity in the following network.

(*ii*) Number the nodes in the following network.

(*iii*) Calculate the earliest expected time and latest allowable times for the activities and the project given below.

(*iv*) Find the critical path for the project.

1 + 1 + 4 + 2



(*Continued*)

5. A television repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs the sets in the order in which they come in and if the arrivals of sets are approximately Poisson with an average rate of 10 per 8 hour a day, then

   (*i*) What is the repairman's idle time each day?

   (*ii*) Find the expected number of units in the system and in the queue.

   (*iii*) What is the probability that there will be no television set waiting in queue for service?

   (*iv*) Find the expected waiting time of a television set being getting the service.

   (*v*) What is the probability that there will be three or more than three television sets in the system? $2 + 2 + 2 + 1 + 1$

6. Derive the difference equation for the (M/M/1 : N/FCFS/∞) queueing system in steady-state condition and hence find the expression for p_n. 8

7. Find the optimum order quantity for a product for which the price breakes are as follows :

Quantity (Units)	Price per unit (Rs.)
$0 < q_1 < 100$	20
$100 \leq q_2 < 200$	18
$200 \leq q_3$	16

Monthly demand for the product is 400 units. Storage cost is 20% of the unit cost of the product, cost of ordering is Rs. 25 per month. 8

[*Internal Assessment* — 10 Marks]

(*Dynamical Meteorology* - I)

Answer **Q.No.1** any **four** from the rest

1. Answer any *two* questions : 2x2

(*a*) What is the concept of weather forecasting ?

(*b*) What is relative humidity ?

(*c*) Define the potential temperature ?

2. (*a*) Deduce the Clausius-Clapeyron equation in the following form

$$\frac{de_s}{dT} = \frac{e_s L}{R_v T^2}$$

Interpret the above equation. 7

(*b*) What do you mean by isobaric cooling process ? 2

3. (*a*) What do you mean by entropy ? What is isentropic process ? Show that for an isentropic process $p\alpha^{\gamma} = \text{constant}$. 2 + 1 + 4

(*b*) What do you mean by Geodynamical Paradox ? 2

4. Drive the barotropic model in the atmosphere in the following form 9

$$\nabla_p^2 \frac{\partial z}{\partial t} = - J(z, gf^{-1} \nabla_p^2 z + f)$$

5. Derive the circulation theorem in the atmosphere in the following form

$$\frac{dC}{dt} = - \oint \frac{dp}{\varrho} + W = 2\Omega \frac{dF}{dt}$$

Interpret each term of the equation. 9

6. Write down the main characteristics of an aerological diagram. Derive the area equivalence of the tephigram and discuss its important properties. 1 + 3 + 5

7. Obtain the atmospheric energy equation and interpret each term. 9

8. Derive the equation of motion of an air parcel in the atmosphere in spherical co-ordinate system. 9

[*Internal Assessment* — 10 Marks]