 PG/IIS/A.MATH/MA - 1205/09

2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Functional Analysis*)

PAPER—MA - 1205

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Answer **Q.No.1, 2** and any *four* from **Q.No.3** to **Q.No.8**

1. Answer any *two* : 2x2

   (*a*) Define contraction mapping on a metric space.

   (*b*) In an inner product space if $(x, u) = (x, v)$ for all x then show that $u = v$.

(c) State open mapping theorem.

2. Answer any *one*: 8

(a) Show that for $p \geqslant 1$ the metric space

$$l_p = \left\{ x = \{x_j\} : \sum_j |x_j|^p < \infty \right\}$$

with metric

$$d(x, y) = \left(\sum_{j=1}^{\infty} |x_j - y_j|^p \right)^{1/p}$$

is complete.

(b) Let (X_1, d_1) and (X_2, d_2) be metric spaces. If $f : X_1 \to X_2$ be continuous and X_1 be compact then show that f is uniformly continuous.

3. Prove that the Volterra Integral Equation

$$f(s) = x(s) - \mu \int_a^s k(s, t)\, x(t)\, dt$$

has a unique solution if $f(t)$ and $x(t)$ are continuous on $[a, b]$ and $k(s, t)$ is continuous on the triangular region $a \leqslant t \leqslant s$, $a \leqslant s \leqslant b$. 7

4. Define a bounded linear operator. Let X and Y be normed linear spaces and $T: X \to Y$ be a linear operator. Prove that T is continuous if and only if T is bounded. 7

5. Let X be a real normed linear space and M be linear subspace of X. If $z \in X$ and dist $(z, M) = d > 0$ then show that there exists a bounded linear functional g such that

$$g(M) = \{0\},\ g(z) = d \text{ and } \|g\| = 1.$$ 7

6. State and prove the closed graph theorem. 7

7. State and prove the Cauchy Schwarz inequality in an inner product space. Prove that inner product is a continuous mapping. 5 + 2

8. (*a*) If $\{e_1, e_2, ..., e_n\}$ is a finite orthonormal set in an inner product space X and x is an element of X then prove that

$$\sum_{i=1}^{n} |(x, e_i)|^2 \leq \|x\|^2.$$ 4

(*b*) If T_1 and T_2 are self adjoint operators then prove that $T_1 T_2$ is self adjoint if and only if $T_1 T_2 = T_2 T_1$. 3

[*Internal Assessment*: 10 Marks]
