M.Sc 1st Semester Examination 2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA - 1102

(*Complex Analysis*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 4** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. (*a*) State Cauchy - Riemann equations. Show by an example that a function

$$f(z) = u(x, y) + iv(x, y)$$

ceases to be differentiable at the point (x_0, y_0) in the domain C even if the Cauchy - Riemann equations are satisfied at that point. 4

(*b*) Let $f(z)$ be continuous on a domain D and let

$$\int_{z_0}^{z_1} f(z)\,dz,\ z_0,\ z_1 \in c,$$

be path independent, then prove that there exists a function $F(z)$ such that

$$F'(z) = f(z)$$

for all z in D. 4

(*c*) When is a function $f(z)$ said to have a pole of order m at z_0? If a function $f(z)$ has a pole of order m at z_0, prove that $\frac{1}{f(z)}$ has a zero of order m at z_0. 4

(*d*) Given $f(z)$ to be analytic, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$$

holds. 4

2. (*a*) Show that under suitable condition, to be stated by you

$$f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)\, dz}{z - a}.$$ 4

(*b*) Expand the function

$$f(z) = \frac{3}{z^2 - 1}$$

in the annulus $1 < |z+2| < 3$. 4

(*c*) Find v such that $f(z) = u + iv$ is analytic where

$$u = e^{-x}(x\sin y - y\cos y).$$

Also find $f(z)$ in term of z. 4

(*d*) Prove that the given function

$$f(z) = \frac{z^8 + z^4 + 2}{(9z^2 + 12z + 4)(z-1)^3}$$

has three singularities and hence name the singularities. 4

3. (*a*) Find the number of zeros of the polynomial

$$z^4 - 5z + 1$$

in the annulus $1 < |z| < 2$. 4

(*b*) Find all the Möbius transformation which transforms the half plane $I(z) \geq 0$ onto the unit circular disc $|w| \leq 1$. 4

(*c*) Evaluate the following by the method of contour integration (any *two*): 4×2

(*i*) $\int_0^\infty \frac{dx}{x^2 + x + 1}$

(*ii*) $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}\, dx,\ a > 0$

(*iii*) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx$

(*iv*) $\int_0^{2\pi} \frac{dx}{3+\cos x}$.

4. Answer the following: 2×4

(*a*) State the necessary and sufficient conditions so that

$$f(z) = u(x, y) + iv(x, y)$$

be analytic in a region.

(*b*) Construct the analytic function $w = f(z)$ if its real part is $e^x \cos y$ and if $f(0) = 1$.

(*c*) Evaluate:

$$\int_{|z|=1} z\bar{z}\, dz.$$

(*d*) Discuss the nature of the singularities of the function

$$\frac{\sin z}{(z-\pi)^2}.$$

[*Internal Assessment* — 10 Marks]
