2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Fuzzy Sets and their Applications and Soft Computing*)

PAPER—2203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP—A

(*Fuzzy Sets and their Applications*)

[*Marks* : 25]

Answer **Q. No. 1** and any *three* from the rest

1. Answer any *two* questions: 1 + 1

(*a*) Define a fuzzy number.

(*b*) Give an example of triangular fuzzy number.

(*c*) Give an example of fuzzy stochastic event.

2. Using characteristic function prove that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

and $A \subset B \subset C$ implies $A \subset C$. 3 + 3

3. What are causes of uncertainty? Explain the traditional and modern view of uncertainty. What are random and non-random uncertainties? 2 + 2 + 2

4. (*a*) Simplify

$$2[3, 4, 7] + 4[1, 2, 3, 4] - 5[8, 10] + 6.$$ 2

(*b*) Let the membership functions of two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ be

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 , & x < 2 \\ x-2 , & 2 \leq x < 3 \\ 4-x , & 3 \leq x < 4 \\ 0 , & x \geq 4 \end{cases}$$

$$\mu_{\underset{\sim}{B}}(x) = \begin{cases} 0 & x < 3 \\ x-3 & 3 \leq x < 4 \\ 5-x & 4 \leq x < 5 \\ 0 & x \geq 5 \end{cases}$$

Find the membership functions of $\underset{\sim}{A} \cup \underset{\sim}{B}$ and $\underset{\sim}{A} \cap \underset{\sim}{B}$. 2 + 2

5. Using Werners approach derive the equivalent crisp LPP for the following fuzzy LPP:

Maximize $Z = Cx$
subject to $(Ax)_i \leq \bar{b}_i$, $i = 1, 2, \ldots, m$

$$x \geq 0$$

where p_i is the tolerance for the ith fuzzy resource for each $i = 1, 2, \ldots, m$. 6

6. Using Zimmermann's method, determine the crisp LPP equivalent to the fuzzy LPP:

Find x

such that $g_0(x) = 13x_1 + 12x_2 \gtrsim \underset{\sim}{b_0}$

$$g_1(x) = 4x_1 + 3x_2 \lesssim \underset{\sim}{b_1}$$

$$g_2(x) = 2x_1 + 5x_2 \lesssim \underset{\sim}{b_2}$$

$$g_3(x) = 3x_1 + 4x_2 \lesssim \underset{\sim}{b_3}$$

$$x_1, x_2 \geq 0$$

where the goal b_0 of the fuzzy objective is 25 and its corresponding tolerance p_0 is 2, and the fuzzy resource b_i and their tolerance p_i are as follows: 6

$b_1 = 12$, $b_2 = 10$, $b_3 = 12$, $p_1 = 1$, $p_2 = 1$ and $p_3 = 2$.

[*Internal Assessment* : 5 Marks]

GROUP—B

(*Soft Computing*)

[*Marks* : 25]

1. Answer any *two* of the following:

(*a*) $\{X_1^T = [2 \quad 2],\ d_1 = 0\}$

$\{X_2^T = [1 \quad -2],\ d_2 = 1\}$

$\{X_3^T = [-2 \quad 2],\ d_3 = 0\}$

$\{X_4^T = [-1 \quad 1],\ d_4 = 1\}$.

Solve it with single vector input, two element perceptron network using initial weights $W(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and initial biases $b(0) = [0]$ for two iterations only. 8

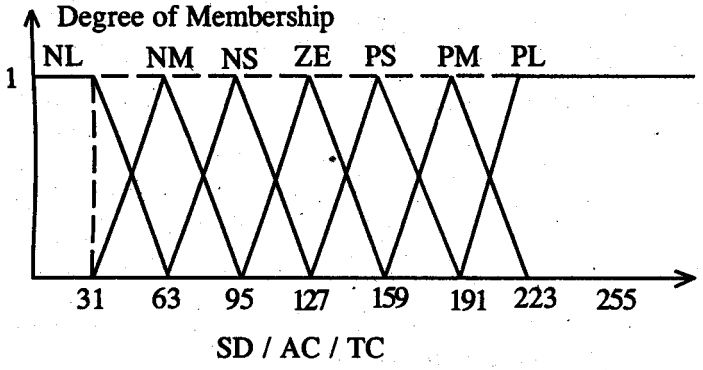
(*b*) A controller is used to maintain a vehicle at a desired speed. The system consists of two fuzzy inputs — speed difference (SD) and

acceleration (AC) and one fuzzy output, Throttle control (TC). The fuzzy rule base for the system is

IF (SD is NL) AND (AC is ZE) THEN (TC is PL)
IF (SD is ZE) AND (AC is NL) THEN (TC is PL)
IF (SD is NM) AND (AC is ZE) THEN (TC is PM)
IF (SD is NS) AND (AC is PS) THEN (TC is PS)
IF (SD is PS) AND (AC is NS) THEN (TC is NS)
IF (SD is PL) AND (AC is ZE) THEN (TC is NL)
IF (SD is ZE) AND (AC is NS) THEN (TC is PS)
IF (SD is ZE) AND (AC is NM) THEN (TC is PM)

where NL: Negative Large
NM: Negative Small,
ZE: Zero
PS: Positive Small
PM: Positive Medium,
PL: Positive Large.

Fuzzy sets for SD, AC and TC (in Normalised form) are



If the normalised speed difference be 100 and the normalised acceleration be 70, then what should be the Throttle control in normalised form. 8

(*c*) Maximize $f(x) = x^3 - 12x^2 + 45x$ using a real-coded Genetic Algorithm in the interval (0, 4) using Roulette wheel selection, Arithmetic cross-over and Random mutation (for only one generation).

[Given that population size = $N = 5$.

Initial population $x(i) = 1{\cdot}852, 3{\cdot}828, 1{\cdot}380, 1{\cdot}472, 1{\cdot}776$ $i = 0, 1, \ldots 4$.

Random nos. to be used for selection: ·46, ·30, ·82, ·90, ·56

" " " " " for cross-over: ·346, ·13, ·982, ·09, ·656

" " " " " for mutation: ·19, ·59, ·65, ·45, ·96

probability of cross-over, $p_c = 0{\cdot}4$

probability of mutation, $p_m = 0{\cdot}2$

λ for Arithmetic cross-over: $\lambda = 0{\cdot}346$

For mutation, r (random no.) = ·55,

Δ (permutation value) = 1·20]. 8

2. Answer any *one* of the following:

(*a*) Give the schematic diagram of the working cycle of GA. Also explain single-point cross-over and two-point cross-over in a Binary-coded GA with examples. 4

Or

Explain the Roulette - Wheel selection for reproduction process in Genetic Algorithm.

(*b*) Mention logical connectives,
Given that

$\tilde{P}$: Rina is efficient; Truth value

of $\tilde{P} = T(\tilde{P}) = 0{\cdot}8$

$\tilde{Q}$: Bimal is efficient; Truth value

of $\tilde{Q} = T(\tilde{Q}) = 0{\cdot}65$

Find the truth values of logical connectives of above propositions $\tilde{P}$, $\tilde{Q}$. 4

Or

Let $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$

$\tilde{A} = \{(a, 0)\ (b, 0{\cdot}8)\ (c, 0{\cdot}6)\ (d, 1)\}$

$\tilde{B} = \{(1, 0{\cdot}2)\ (2, 1)\ (3, 0{\cdot}8)\ (4, 0)\}$

Determine the implication relation IF x is $\tilde{A}$ then y is $\tilde{B}$.

(*c*) Make a comparison between the Biological and Artificial Neural Networks. Describe the architecture of a single layer feed forward network. 4

Or

Generate the output of logical AND function by a single perceptron using initial weight = $W = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and initial bias $b = -2$.

[*Internal Assessment* : 5 Marks]