2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER — MA - 2204

Full Marks : 50

Time : 2 hours

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Non linear Optimization*)

Q.No.1 is compulsory and any *three* from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* of the following : 2 x 5

   (*a*) Define : 'Degree of difficulty'.

   (*b*) Define : "Nash Equilibrium Strategy and Nash Equilibrium Outcome".

(*Turn Over*)

(*c*) Give the definition of quadratic programming problem including mathematical form.

(*d*) Under what conditions, the Kuhn-Tucker condition for quadratic programming problem will be necessary and sufficient ?

(*e*) What do you mean by differentiable concave function ?

(*f*) Define : Fritz John Saddle Point problem (FJSP).

2\. (*a*) State and prove Fritz John Saddle Point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz John Saddle Point problem ?

(*b*) Write down the basic steps of Frank Woffe algorithm including stopping criteria for linearly constrained convex programming problem (which can be considered by you). 7 + 3

3\. (*a*) Derive the Kuhn-Tucker conditions for quadratic programming problem.

(*b*) State and prove the sufficient optimality criteria of the solution, on minimization problem with differentiability.

(*c*) State Farka's theorem. $5+3\frac{1}{2}+1\frac{1}{2}$

4. (*a*) Prove that a pair $\{y^*, z^*\}$ constitutes a mixed-strategy Nash equilibrium solution to a bimatrix game (A, B) if, and only if, there exists a pair (p^*, q^*) such that $\{y^*, z^*, p^*, q^*\}$ is a solution of the following bilinear programming problem :

$$\min_{y,\, z,\, p,\, q} \; [y'Az + y'Bz + p + q]$$

subject to $\quad Az \geqslant -p\,l_m, \quad B'y \geqslant -q\,l_m$

$$y \geqslant 0,\ z \geqslant 0,\ y'l_m = 1,\ z'l_n = 1.$$

(*b*) Describe briefly the Beale's method for solving quadratic programming problem. $4+6$

5. (*a*) Briefly describe about the constraint qualifications in case of saddle point problem without differentiability.

(*b*) Min $f(X) = c_1 x_1 x_2 x_3 + c_2 x_1 x_2^{-1}$

$+ c_3 x_2 x_3^{-2} + c_4 x_1^{-3} x_3$.

Find the dual problem of this primal problem and solve this problem in terms of c_1, c_2, c_3 and c_4, using Geometric Programming Method.

5+5

6. (*a*) Use the chance constrained programming technique to find an equivalent deterministic non-linear programming problem to the following stochastic programming problem :

$$\text{Minimize} \quad F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \quad P\left[\sum_{j=1}^{n} a_{ij} x_j \leq b_i\right] \geq p_i$$

$$x_j \geq 0, \; i = 1, 2, \ldots, n$$
$$j = 1, 2, \ldots, n$$

where a_{ij} are random variables and p_i are specified probabilities.

(*b*) Define :

(*i*) The (primal) quadratic minimization problem.

(*ii*) The quadratic dual maximization problem. 7 + 3

[*Internal Assessment* : 10 Marks]

(*Dynamical Oceanology* - II)

Answer any *four* questions

The questions are of equal value

1. Define absolute vorticity. Deduce the general vorticity equation.

2. Deduce the equations of thermal wind. Hence deduce the Taylor-Proudman theorem.

3. Show that the plane Rossby waves moves West ward with a speed $\frac{\sigma}{k}$ depending only on the wave number $K = \sqrt{k^2 + l^2}$.

(with usual notations).

4. Deduce the Sverdrup equations as

$$\frac{\partial P}{\partial x} = f M_y + \tau_x \quad \text{and} \quad \frac{\partial P}{\partial y} = -f M_x + \tau_y$$

(with usual notations).

5. Obtain the solution of the equation of motion for the pure wind drift current in plane homogeneous layer of fluid as the infinite depth rotating uniformly about a vertical axis.

6. In two-dimensional model of ocean current, solve the problem of viscous boundary layer and show that weak back flow appears close to the external edge of the boundary layer.

[*Internal Assessment* : 10 Marks]