2009

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Differential Geometry and Magnetohydrodynamics*)

PAPER—MA 2202

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP — A

(*Differential Geometry*)

[*Marks* : 25]

(*Turn Over*)

Answer any *two* questions

1. (*a*) Define rectifiable curve in Euclidean space $\mathbb{R}^n$. 2

(*b*) Show that the plane curve given by

$$\vec{\gamma} = (\gamma_1(t), \gamma_2(t))$$

where

$$\left.\begin{aligned} \gamma_1(t) &= t \\ \gamma_2(t) &= t\cos\frac{1}{t} \quad \text{for } 0 < t \leq 1 \\ &= 0 \quad\quad\;\; \text{for } t = 0 \end{aligned}\right\} (0 \leq t \leq 1)$$

is not rectifiable. 6

(*c*) What do you mean by allowable change of parameter for a regular curve? 2

2. (*a*) State the Serret-Frenet equations for a space curve. 2

(*b*) If $\vec{\gamma} = \vec{\gamma}(t)$ is an arbitrary representation of a space curve of class ≥ 2, then show that the curvature can be given by 3

$$|k| = \frac{\| \vec{\gamma}' \times \vec{\gamma}'' \|}{\| \vec{\gamma}' \|^3}.$$

(*c*) Show that for a curve lying on a sphere of radius a and such that the torsion τ is never zero, the following equation is satisfied

$$\left(\frac{1}{K}\right)^2 + \left(\frac{\dot{K}}{K^2 \tau}\right)^2 = a^2.$$

[Symbols have usual meaning]. 5

3. (*a*) If S is a connected simple surface and if P and Q are arbitrary points on S, then show that there exists a regular arc connecting P and Q. 4

(*b*) Define second fundamental form on a surface of class $\geqslant 2$. 3

(*c*) Show that the surface given by

$$\vec{x} = (u,\ v,\ u^2 + v^3),\ u,\ v \in \mathbb{R}$$

is elliptic when $v > 0$, hyperbolic when $v < 0$, and parabolic for $v = 0$. 3

[*Internal Assessment* : 5 marks]

GROUP—B

(*Magnetohydrodynamics*)

[*Marks* : 25]

Answer any *two* questions

1. Derive the equation for the magnetic induction in magnetohydrodynamic flows and explain the significance of flows at high and low magnetic Reynolds number. 6 + 2 + 2

2. State and prove Alvén's theorem. 2 + 8

3. A viscous incompressible finitely conducting fluid flows steadily under a uniform pressure gradient in a channel formed by two infinite parallel plates which are non-conducting. If a uniform magnetic field acts perpendicular to the channel walls, find the velocity and magnetic field in the channel. 10

[*Internal Assessment* : 5 marks]
