M.Sc. 2nd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Stochastic Process and Regression*)

PAPER—MTM - 206

Full Marks : 25

Time : 1 hours

Answer **Q. No.1** and any **two** from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *two* questions : 2 × 2

   (*a*) Define Markov chain and give an example of this chain. 1 + 1

   (*b*) If $r_{12\cdot3} = 1$, show that $R_{1\cdot23} = 1$ 2

(*Turn Over*)

(*c*) Define 'persistent state' and 'transient state' in a Markov Chain with discrete state space and discrete parameter (time). 2

2. Let $\{X_n : n \geq 0\}$ be a Markov chain having the state space $S = \{1, 2, 3, 4\}$, where transition probability matrix is given by 8

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

Identify the states as transient, persistent and ergodic.

3. Described the pure birth process and discuss Yule-Furry process involving in this process and hence solve it. 8

4. (*a*) Using an appropriate subscript notation, deduce the regression equation of X_1 on X_2, X_3 and X_4.

(*b*) Find the stationary probability distribution of a Markov chain with two states $S = \{0, 1\}$ and transition matrix 5 + 3

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}.$$

[*Internal Assessment* : 5 Marks]