M.Sc. 2nd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Abstract Algebra and Linear Algebra*)

PAPER—MTM-203

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

GROUP – A

(*Abstract Algebra*)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and any **two** questions from the rest

1. Answer any *two* questions : 2×2

(*a*) Define kernel of a group homomorphism and explain it with an example. 2

(Turn Over)

(*b*) Show that characteristic of any two non-zero elements of an integral domain are same. 2

(*c*) Define simple group and give an example of it. 2

2. (*a*) State and prove the fundamental theorem of group homomorphism. 5

(*b*) Show that direct product $z \times z$ is not a cyclic group. 3

3. (*a*) If G be a group of order p^n, where p is a prime number and n be any positive integer, show that center of G is a non-trivial group. 5

(*b*) Show that a field has no non-trivial proper ideals. 3

4. (*a*) If R be a commutative ring with unity and $S \neq R$ be an ideal of R, show that R/S is an integral domain if and only if S is a prime ideal of R. 5

(*b*) Show that every field is an Euclidean domain. 3

[*Internal Assessment* : 5 Marks]

GROUP – B

(*Linear Algebra*)

[*Marks* : 25]

Time : 1 hour

Answer Q. No. 5 and any two questions from the rest

5. Answer any *two* questions : 2 × 2

(*a*) Let V be a finite dimensional vector space. What are the characteristic and minimal polynomials for the zero operator on V?

(*b*) If dim $V = m$, then find dim $L(V, V)$, where $L(V, V)$ denotes the space of all linear transformations from V to V.

(*c*) Let T be a linear operator on a vector space $V(F)$. If $T^2 = 0$. (0 being zero transformation), what can you say about the relation of the range of T to the null space of T?

(*d*) Define modular lattice with an example.

6. (*a*) Let $T_a : \mathbb{R}^3 \to \mathbb{R}^3$ be a map given by $T_a(x, y, z) = (x, ay, z)$, $a \in \mathbb{R}$ being fixed. Show that T_a is an isomorphism. What can you say about T_0? 4

(*b*) Give the definition of lattice with respect to poset and also give the definition of lattice with respect to algebra. Show that the two definitions are equivalent. 4

7. (*a*) Let T be a linear operator on a finite dimensional vector space V. Prove that the following are equivalent : 4

(*i*) c is a characteristic value of T

(*ii*) the operator T−CI is singular (non-invertible)

(*iii*) $\det(\mathrm{T} - \mathrm{CI}) = 0$.

(*b*) Prove that a chain is a distributive lattice. 2

(*c*) Let $T : \mathbb{R}^3 \to \mathbb{R}$ be linear. Show that there exist scalars a, b and c such that

$$T(x, y, z) = ax + by + cz$$

for all $(x, y, z) \in \mathbb{R}^3$. Can you generalise this result for $T : F^n \to F$? Justify your answer. 2

8. (*a*) Find the minimal polynomial for the real matrix. 4

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}.$$

(*b*) Is there a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? Justify your answer. 2

(*c*) Define $T : P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ by

$$T(f) = \begin{pmatrix} f(1) & f(2) \\ f(3) & f(4) \end{pmatrix}$$

Examine whether T is invertible or not. 2

[*Internal Assessment* : 5 Marks]