M.Sc. 2nd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Numerical Analysis*)

PAPER—MTM-202

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) Prove that

$$\mu\delta f(x) = \left(\frac{\Delta E^{-1} + \Delta}{2}\right) f(x)$$

where the symbols have their usual meanings.

(*b*) Express the polynomial

$$10x^3 - 12x^2 + 100x + 5$$

interms of Chebyshev polynomials.

(*c*) Explain stable, unstable and conditionally stable iteration schemes.

(*d*) What do you mean by ill-conditioned system of linear algebraic equations ?

(*e*) What is a spline of order 3 ?

(*f*) Evaluate the integral

$$I = \int_{-1}^{1} (1 - x^2)^{3/2} \cos x \, dx$$

using Gauss-Chebyshev three-point quadrature formula.

2. (*a*) Deduce Stirling's central difference interpolation formula. 8

(*b*) Describe least square method to approximate a function $y = f(x)$ with the help of orthogonal polynomials. What is the advantage to use orthogonal polynomials than other polynomials ? 6 + 2

3. (*a*) Describe power method to find the largest (in magnitude) eigenvalue and the corresponding eigenvector of a matrix. Can this method be used to find the least eigenvalue ? Explain. 6 + 2

(*b*) Describe Milne's predictor-corrector method to solve the differential equation

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$ 8

4. (*a*) Establish $(n+1)$ point Gauss-Legendre quadrature formula for the integral

$$\int_a^b f(n)\,dx.$$

Obtain the values of C_0, C_1 and x_1 so that the quadrature rule

$$\int_0^1 f(x)\,dx = C_0 f(0) + C_1 f(x_1)$$

is exact for polynomial of the highest possible degree. What is the degree ? 5 + 3 (*b*) Derive an explicit finite difference scheme for solving the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, c > 0, 0 < t < T, 0 < x < a$$

with initial and boundary conditions

$$u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = g(x), 0 < x < a$$

and $u(0, t) = \phi(t)$, $u(a, t) = \Psi(t)$, $t > 0$ respectively, such that the error is of quadratic order. 8

[*Internal Assessment* : 10 Marks]