Total Pages—5 **PG/IS/MTM-103/13**

M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Ordinary Differential Equations and Special Functions*)

PAPER—MTM-103

Full Marks : 50

Time : 2 hours

Answer Q. No. 1 and any three from Q. No. 2 to Q. No. 5

The figures in the right-hand margin indicate marks

1. Answer any *five* questions : 2×5

(*a*) Show that $J_n(z)$ is an odd function of z if n is odd.

(*b*) Define fundamental set of solutions and fundamental matrix for system of differential equation.

(*Turn Over*)

(*c*) What do you mean by Wronskian in ODE and its utility ?

(*d*) Define orthogonal functions associated with Strum-Liouville problem.

(*e*) What do you mean indicial equation concerning on ODE ?

(*f*) Let $P_n(z)$ be the Legendre polynomial of degree n. If

$$1+z^5=\sum_{n=0}^{5} C_n P_n(z),$$

then find the value of C_5

(*g*) When a boundary problem is a Strum-Liouville problem ?

2. (*a*) Find the general solution of the equation

$$2z(1-z)\,w''(z)+w'(z)+4w(z)=0$$

by the method of solution in series about $z=0$, and show that the equation has a solution which is polynomial in z. 6

(*b*) Show that

$$n\,P_n(z) = z\,P_n'(z) - P_{n-1}'(z),$$

where $P_n(z)$ denotes the Legendre polynomial of degrees n. 4

3. (*a*) Find the characteristics values and characteristic functions of the Sturm-Liouville problem.

$$(x^3 y')' + \lambda x y = 0;\ \ y(1) = 0,\ \ y(e) = 0.$$ 5

(*b*) Determine whether the matrix

$$B = \begin{pmatrix} e^{4t} & 0 & 2e^{4t} \\ 2e^{4t} & 3e^{t} & 4e^{4t} \\ e^{4t} & e^{t} & 2e^{4t} \end{pmatrix}.$$

is a fundamental matrix of the system $\frac{dX}{dt} = AX$ where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ A = \begin{pmatrix} 1 & -3 & 9 \\ 0 & -5 & 18 \\ 0 & -3 & 10 \end{pmatrix}.$$ 5

4. (*a*) Solve the following boundary value problem using Green's function

$$\frac{d^2y}{dx^2} + y = x^2; \;\; y(0) = y(\pi/2) = 0. \qquad 7$$

(*b*) Prove that

$$\frac{d}{dz}\left[z^{-n} J_n(z)\right] = -z^{-n} J_{n+1}(z),$$

where $J_n(z)$ is the Bessel's function. 3

5. (*a*) If the vector functions $\phi_1, \phi_2, ..., \phi_n$ defined as follows :

$$\phi_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \cdot \\ \phi_{n1} \end{bmatrix}, \; \phi_2 = \begin{bmatrix} \phi_{12} \\ \phi_{22} \\ \vdots \\ \phi_{n2} \end{bmatrix}, \; .... \phi_n = \begin{bmatrix} \phi_{1n} \\ \phi_{2n} \\ \vdots \\ \phi_{nn} \end{bmatrix}$$

be n solutions of the homogeneous linear differential equation $\frac{dx}{dt} = A(t)\, x(t)$ in the interval $a \le t \le b$, then these n solutions are linearly independent in $a \le t \le b$ iff Wronskian $W[\phi_1, \phi_2, ...\phi_n] \ne 0 \quad \forall t$, on $a \le t \le b$. 5

(*b*) Find the Legendre polynomial $P_n(z)$ by solving the following differential equation

$$(1-z^2)\frac{d^2w}{dz^2} - 2z\frac{dw}{dz} + n(n+1)w = 0.$$ 5

[*Internal Assessment* : 10 Marks]