Total Pages—4 **PG/IS/MTM-101/13**

M.Sc. 1st Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Real Analysis*)

PAPER—MTM-101

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **four** from **Q. No. 2** to **Q. No. 7**

The figures in the right-hand margin indicate marks

1. Answer any *four* questions : 2 × 4

(*a*) Define cover, open cover in metric space. Give an open cover of (0, 1).

(*b*) If $f(x) = 4x^2 + 5$ and $g(x) = 8$ then find the RS-integral

$$\int_1^4 f(x)\,dg(x)$$

(*Turn Over*)

(*c*) Let $f : [a, b] \to \mathbb{R}$ be a function of bounded variation on $[a, b]$ and $[c, d] \subset [a, b]$. Prove that f is a function of bounded variation on $[c, d]$.

(*d*) Let $f : [a, b] \to \mathbb{R}$ be a continuous function. "Identify" $f([a, b])$.

(*e*) Show by an example that closed and bounded subset of a metric space is not necessarily compact.

2. (*a*) Show that a metric space (X, d) is connected if and only if every continuous function $f : X \to \{0, 1\}$ is constant, where the two point set $\{0, 1\}$ is considered to be a metric space under discrete metric. 4

(*b*) Verify wheather the following sets are connected and compact sets : 4

(*i*) Set of all rationals of $\mathbb{R}$.

(*ii*) $\{(x, y) \in \mathbb{R}^2 : xy \neq 0 \text{ and } x^2 + y^2 = 5\}$.

3. (*a*) Prove that the necessary and sufficient condition for a function to be of bounded variation on $[a, b]$ is that it can be expressed as the difference between two monotonic function on $[a, b]$. 5

(*b*) Find the variation function for the function $f(x) = [x] - x, x \in [0, 2]$. 3

4. (*a*) Show that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$, there exists a partition P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$. 5

(*b*) Evaluate the RS-integral

$$\int_{-2}^{3} 2x^4 \, d\left(2|x| + 3x^2 - 5\right).$$ 3

5. (*a*) Define a measurable function on $[a, b]$. Prove that any continuous function defined over $[a, b]$ is always measurable on $[a, b]$. 1 + 3

(*b*) Let $\{E_n\}$ be a sequence of measurable sets such that $E_{n+1} \subseteq E_n$ for each $n \in \mathbf{N}$. If $\mu(E_1)$ is finite, then show that

$$\mu\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} \mu(E_n)$$ 4

6. (*a*) Let f be a non-negative measurable function on X and E be a measurable subset of X be such that

$$\int_E f d\mu = 0.$$

Then show that $f = 0$ a.e. $\mu(x)$ on E. 3

(*b*) If $f_1, f_2 \in L^1(\mu)$ then show that $f_1 + f_2 \in L^1(\mu)$ and $\int (f_1 + f_2) d\mu = \int f_1 \, d\mu + \int f_2 \, d\mu$. 5

7. (*a*) Define Lebesgue integral for unbounded measurable function on $[a, b]$. 2

(*b*) Let $f(x)$ be defined on $[0, 1]$ as

$$f(x) = \frac{1}{x^{4/5}}, \quad 0 < x \le 1$$
$$= 0 \quad , \quad x = 0.$$

Show that $f(x)$ is Lebesgue integrable on $[0, 1]$. Find the value of the integral. 6

[*Internal Assessment* : 10 Marks]