PG/IIIS/MTM-306/13

M.Sc. 3rd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Operational Research Modelling* - I / *Dynamical Meteorology* - I)

PAPER—MTM-306

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Operational Research Modelling* - I)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *four* questions : 2×4

(*a*) What is the basic principle of dynamic programming ? Explain stage and state.

(*b*) Define various costs related to inventory control models.

(*c*) Discuss 'steady state' and 'transient state' in a queueing system.

(*d*) What are the differences between PERT and CPM ?

(*e*) What are the advantages to use simulation to solve a problem ?

(*f*) Explain the 'individual' and 'group' replacement policies. Give the expression of average failure following mortality theorem in the case of Individual Replacement Policy.

2. Use dynamic programming method to solve the following LPP : 8

$$\text{Maximize } Z = 8x_1 + 7x_2$$
$$\text{subject to } 2x_1 + x_2 \leq 8$$
$$5x_1 + 2x_2 \leq 15$$
$$x_1,\ x_2 \geq 0$$

3. A small project consists of seven activities. The details of which are given below : 8

Activity	Duration (in days)			Immediate predecessor activity
	Most likely	Optimistic	Pessimistic	
A	3	1	10	—
B	6	2	14	*A*
C	3	3	3	*A*
D	10	4	22	*B, C*
E	7	3	15	*B*
F	5	2	14	*D, E*
G	4	4	4	*D*

(*i*) Draw the network.
(*ii*) Find the critical path.
(*iii*) Find the expected project completion time.
(*iv*) Find the next most critical path.

4. A firm has a single channel service station with the following arrival and service time probability distributions :

Inter-arrival time (minutes)	Probability	Service time (Minutes)	Probability
10	0·10	5	0·08
15	0·25	10	0·14

Inter-arrival time (minutes)	Probability	Service time (Minute)	Probability
20	0·30	15	0·18
25	0·25	20	0·24
30	0·10	25	0·22
		30	0·14

The customer's arrived at the service station is a random phenomenon and the time between the arrival varies from 10 minutes to 30 minutes. The service time varies from 5 minutes to 30 minutes. The queuing process begins at 10 am and proceeds for nearly 8 hours. An arrival goes to the service facility immediately, if it is free. Otherwise, It will wait in a queue. The queue discipline is first-come-first-served. If the attendant's wages are Rs. 10 per hour and the customer's waiting time costs Rs. 15 per hour, then would it be an economical proposition to engage a second attendant ? Answer using Monte Carlo simulation technique. 8

5 Find the economic lot size for the inventory model with finite replenishment rate, shortages are allowed but fully backlogged, uniform finite demand and zero lead time. So that the total average cost is minimum. 8

6. Derive and solve the steady state difference equations governing the queuing model (M/M/1) : (∞/FIFO/∞). 8

7. The following mortality rates have been observed for a certain type of fuse :

Week	:	1	2	3	4	5
Percentage failing by the end of work	:	5	15	35	57	100

There are 1000 fuses in use and it costs Rs. 5 to replace an individual fuse. If all fuses were replaced simultaneously it would cost Rs. 1·25 per fuse. It is proposed to replace all fuses at fixed intervals of time, whether or not they have burnt out and to continue replacing burnt items as they fail. At what time intervals should the group replacement be made ? Is group replacement policy is better than the individual replacement policy ? Justify. 8

8. The following mortality rates have been observed for a certain type of light bulb :

Week	1	2	3	4	5
Percentage of failing at the end of week	10	25	50	80	100

There are 1000 bulbs in use and its cost Rs. 10·00 to replace an individual bulb which has burnt out, if all bulbs were replaced simultaneously, it would cost Rs. 5·00 per bulb. It is proposed to replace all bulbs at fixed intervals, whether or not they have burnt and to continue replace burnt bulbs as they fail. At what intervals should all the bulbs be replaced? 8

[*Internal Assessment* : 10 Marks]

(*Dynamical Meteorology* - I)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *two* questions : 2 × 2

(*a*) What do you mean by entropy and isentropic process ?

(*b*) Explain geo-dynamical paradox.

(*c*) Define potential temperature.

2. (*a*) What is planetary vorticity ? Derive the vorticity equation of an air parcel in the atmosphere and interpret each term.

(*b*) Prove that $T_V = T(1 + 0{\cdot}61\, r)$ where T_V be the virtual temperature, T is temperature and r be the mixing ratio of an air parcel. 7 + 2

3. (*a*) How the thermal wind is formed in the atmosphere ? Derive the thermal wind components in the atmosphere.

(*b*) Derive the relation between the specific heat constant at constant pressure for moist air and dry air. 7 + 2

4. Derive the circulation theorem for an air parcel in the atmosphere and interpret each term in the equation. 9

5. Derive the relation from which saturation temperature can be obtained when saturation of air will be done by adiabatic ascent and for this case, estimate the height at which saturation will attend. 9

6. (*a*) Find the relation between mixing ratio and specific humidity.

(*b*) Obtain the atmospheric energy equation in the atmosphere and interpret each term. 2 + 7

7. (*a*) Derive the expression for final temperature of any mixture of two air masses due to adiabatic isobaric horizontal mixing without condensation.

(*b*) Derive the equation of momentum of an air parcel in the Cartesian co-ordinate. 2 + 7

8. (*a*) What do you mean by isobaric cooling ? Show that relative increase in dew-point temperature is about 5% of the sum of relative increase in mixing ratio and pressure.

Also find a rough estimate of the difference between temperature and dew-point as a function of relative humidity.

(*b*) Derive the hydrostatic equation in the atmosphere. 7 + 2

[*Internal Assessment* : 10 Marks]
