PG/IIIS/MTM-303/13

M.Sc. 3rd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Operations Research/Dynamical Oceanology and Meteorology*)

PAPER—MTM-303

The figures in the right-hand margin indicate marks

(*Operations Research*)

[*Marks* : 50]

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

1. Answer any *four* questions : 2 × 4

(*i*) What do you mean by zero-one integer programming problem ? Give an example.

(*ii*) What is the benefit of using revised simplex method to solve a linear programming problem ?

(*iii*) Explain the application of post-optimal analysis in optimization theory ?

(*iv*) Define order level lot size system.

(*v*) Which optimization procedure do you perfer to solve a NLPP with inequality constraints and why ?

(*vi*) How do you decide a departing variable in a revised simplex method ?

2. (*a*) State Bellman's principle of optimality. Hence

$$\text{Minimize} \quad Z = x_1^2 + x_2^2 + x_3^2$$
$$\text{S.T.} \quad x_1 + x_2 + x_3 \geq 15$$
$$x_1, x_2, x_3 \geq 0$$

8

(*b*) Find the minimum of

$$f(x) = (x_1 + 1)^2 + (x_2 - 2)^2$$

subject to $g_1(x) = x_1 - 2 \leq 0$
$g_2(x) = x_2 - 1 \leq 0$
$x_1 \geq 0, x_2 \geq 0$

using K-T conditions. 8

3. (*a*) Formulate the basic economic order quantity (EOQ) model with constant demand, infinite replenishment rate and zero lead time without shortages to obtain the optimal lotsize and total cost. Deduce the limitation of EOQ formula. 8 + 3

(*b*) An oil engine manufacture purchases 42 lubricants pieces from a vendor. The requirement of these lubricants is 1800 per year. What should be the order quantity per order if the cost per placement of an order is \$ 16 and inventory carrying cost per dollar per year is \$0·20. 5

4. (*a*) Solve the LPP by revised simplex method :

$$\text{Max } Z = 2x_1 + 3x_2 - x_3 + 4x_4 + x_5 - 3x_6$$

subject to

$$x_1 - 2x_2 + x_4 + 4x_5 + \frac{1}{2}x_6 \le 10$$

$$x_1 + x_2 + 3x_3 + 2x_4 + x_5 - x_6 \le 16$$

$$2x_1 + \frac{1}{2}x_2 - x_3 - x_4 + 2x_5 + 5x_6 \le 8$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

8

(*b*) Solve the following integer programming problem by enumeration and rounding method :

$$\text{Max } Z = 6x_1 + 5x_2$$

subject to the constraints

$$2x_1 + 4x_2 \leq 8$$
$$4x_1 + 2x_2 \leq 8$$
$$x_1, x_2 \geq 0 \text{ and integers.}$$ 8

5. (*a*) Obtain Gomory's cutting plane algorithm for the solution of a pure integer programming problem. 8

(*b*) Solve the LPP

$$\text{Max } Z = 3x_1 + 5x_2$$

subject to the constraints

$$x_1 + x_2 \leq 1$$
$$2x_1 + 3x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

Obtain the variations in cost vectors which are permitted without changing the optimal solutions. 8

[*Internal Assessment* : 10 Marks]

(*Dynamical Oceanology and Meteorology*)

[*Marks* : 50]

Time : 2 hours

Answer any **five** questions including **Q. No. 1**

1. (*a*) Define the co-efficient of thermal expansion, the co-efficient of isothermal compression and the co-efficient of adiabatic compression. 3

(*b*) Prove that $K_\eta = K_T - \Gamma\alpha$. Hence deduce that $K_T > K_\eta$. 5

2. State and prove the necessary conditions of thermodynamical equilibrium of a finite volume of sea-water. 8

3. Define Brunt-Väisälä frequency. Explain it in terms of the speed of the sound in the sea-water. 8

4. Prove that the air parcel is stable if $\Gamma_d > \gamma$, neutral if $\Gamma_d = \gamma$ and unstable if $\Gamma_d < \gamma$. 8

5. Define specific entropy and potential temperature. Establish the relation between the specific entropy and the potential temperature. 8

6. Define the virtual temperature of moist air. Prove that $T^* > T$, where T^* and T are respectively the virtual temperature and the temperature of the moist air. 8

7. (*a*) Show that the mixing ratio and the specific humidity of a sample of water vapour are nearly same. 4

   (*b*) Define relative humidity. Give the significances of it. 4

[*Internal Assessment* : 10 Marks]