PG/IIIS/MTM-302/13

M.Sc. 3rd Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Integral Transform and Integral Equations*)

PAPER—MTM-302

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* questions of the following : 2×5

(*a*) If $\bar{f}(k, l)$ be the two-dimensional Fourier transform of a function $f(x, y)$, then what is the Fourier inversion formula to get $f(x, y)$ from $\bar{f}(k, l)$?

(*b*) Define finite Hankel transform of order n of a function $f(r)$, $0 \le r \le a$ and state its inversion formula.

(*Turn Over*)

(*c*) If $F(p)$ denotes the Laplace transform of the function $f(t)$, $t \geq 0$, state the conditions which $f(t)$ must satisfy so that $F(p)$ exists.

(*d*) Define Mellin transform of a function. Find the Mellin transform of sin Kx.

(*e*) What do you mean by Fredholm alternative in integral equation ?

(*f*) When a kernel $k(x, t)$ of an integral equation is said to be degenerated ?

2. (*a*) Find Fourier transform of the function,

$$f(x) = 1, \quad \text{for } |x| \leq 1,$$
$$= 0, \quad \text{for } |x| > 1.$$

Hence evaluate

$$\int_0^{\infty} \frac{\sin x}{x} dx.$$ 4

(*b*) Discuss the solution procedure of non-homogeneous integral (Fredholm) equation when the Kernel is separable.

Utilize this to solve the following integral equation:

$$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)y(t)\, dt$$ 6

3. (*a*) Find the solution of the following problem of free vibration of a stretched string of an infinite length :

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial + 2} = 0, \quad -\infty < x < \infty,$$

BCS, $u(x,0) = f(n)$

$$\frac{\partial}{\partial t} u(x, 0) = g(n)$$

u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 6

(*b*) Use the finite Hankel transform to

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{n^2}{r^2} f,$$

where $f(r)$ is a function of r defined in the interval $(0, a)$, restricting n to the case $n \geq 0$. 4

4. (*a*) Solve the following ODE by Laplace transform technique :

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=e^x+x+1$$

subject to the conditions $y(0)=y'(0)=y''(0)=0$. 4

(*b*) Find the resolvent kernel of the following integral equation and hence find its solution :

$$\varphi(x)=\int_0^x (t-x)\,\varphi(t)\,dt+1.$$ 6

5. (*a*) Use convolution theorem to find the function whose Laplace transform is

$$\frac{p}{(p^2+a^2)^2}.$$ 4

(*b*) State and prove convolution type theorems (both) concerning on Mellin transform. 3

(*c*) Find the solution of the integral equation,

$$\frac{1}{\sqrt{\pi}} \int_0^x \frac{\varphi(t)}{\sqrt{x-t}}\, dt = f(x),$$

by the use of Laplace transform, where $f(x)$ is a given function of x. 3

[*Internal Assessment* : 10 Marks]

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