PG/IVS/MTM-405/13

M.Sc. 4th Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Operational Research Modelling* - II/OM)

PAPER—MTM-405

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

(*Operational Research Modelling*-II)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and *two* from the rest

1. Answer any *two* questions : 2×2

(*a*) Define reliability of an item. Find a suitable expression to find the reliability of an item.

(*b*) Define 'Sequencing' and discuss the principal assumptions in sequencing problems.

(*Turn Over*)

(*c*) Explain how to measure information provided by two events, dependent or independent.

2. (*a*) Describe a method to process *n* jobs through 3 machines in minimum amount of time.

(*b*) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information (Processing time on machines in given in hours and passing is not allowed). 4 + 4

	Jobs						
	I	II	III	IV	V	VI	VII
Machine A_1 :	3	8	7	4	9	8	7
Machine A_2 :	4	3	2	5	1	4	3
Machine A_3 :	6	7	5	11	5	6	12

3. (*a*) Define joint and conditional entropies. Prove that $H(X, Y) \leq H(X) + H(Y)$, with equality iff X and Y are independent. 4

(*b*) Considering the source $S = \{s_1, s_2, s_3\}$ with $p(s_1) = \frac{1}{2}$ and $p\{s_2\} = p\{s_3\} = \frac{1}{4}$, find $H(S^2)$. 4

4. (*a*) Describe the Euler's equation for extremizing a functional of a single variable and deduce the other forms in which it can be expressed. 4

(*b*) An electrochemical system is characterised by the differential equation

$$\frac{dx_1}{dt} = -x_1 + u, \quad \frac{dx_2}{dt} = x_1$$

where u is the control variable chosen so as to minimize the cost functional

$$\int_0^\infty (x_2^2 + 16u^2/3)dt.$$

Show that if the state variables satisfy $x_1(0) = a$, $x_1(\infty) = 0$, $x_2(0) = b$, $x_2(\infty) = 0$, then the optimum choice of u is

$$u(t) = -0{\cdot}366\, x_1(t) - 0{\cdot}433\, x_2(t).$$ 4

[*Internal Assessment* : 5 Marks]

(*Dynamical Meteorology* - II)

[*Marks* : 25]

Time : 1 hour

Answer **Q. No. 1** and any *two* from the rest

1. Answer any *one* question : 2 × 1

(*a*) What do you mean by global circulation ?

(*b*) Show that in a geostropic wind field, an ideal front is necessarily stationary.

2. Derive the general equation of horizontal motion of air in the atmosphere including the effect of frictional forces resulting from turbulent air motion according to the Prandtl theory. 9

3. (*a*) Derive the relation between pressure difference at the top and bottom of a hurricane. 2

(*b*) Show that the tangential velocity of a hurricane varies with the altitude and hence prove that it has a warm core. 5

(*c*) Suppose at a radius of 40 km, tangential velocity decreases from 15 m/s at the surface to 10 m/s at 10 km altitude. Find the radial temperature gradient at a latitude where Coriolis parameter is 0·00005/s and assume 2

$$\frac{g}{T} = 0 \cdot 0333 \text{ m/s}^2/\text{K}$$

4. (*a*) Derive the meridional temperature gradient due to global circulation and also find meridional temperature and temperature gradient at 45°N latitude at sea level. 6

(*b*) What is Rossby wave ? Find the variation of Coriolis parameter to create Rossby wave. 3

[*Internal Assessment* : 5 Marks]