M.Sc. 4th Semester Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Non-linear Optimization / Dynamical Oceanology* - II)

PAPER—MTM-404

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

MTM-404(OR)

(*Non-linear Optimization*)

Q. No. 1 is compulsory and answer any **three** from the rest

1. Answer any *five* from the following : 2×5

(*a*) How is the degree of difficulty defined for a geometric programming problem ? Calculate the degree of difficulty of the following function

$$f = 4 + 2x_1^2x_2^{-1} + 3x_2^{-4} + 5x_1^{-1}x_2^3$$

(Turn Over)

(*b*) Define Nash equilibrium solution and Nash equilibrium outcome in mixed strategies for bimatrix game.

(*c*) Define differentiable concave vector function.

(*d*) What do you meant by 'Theorems of the alternative" in connection with non-linear programming ?

(*e*) What is stochastic programming problem ? Write two important methods for solving stochastic programming problem.

(*f*) What is the necessity of studying non-linear programming problem ?

(*g*) Write down the differences between the quadratic and non-linear programming problems.

2. (*a*) Prove that all strategically equivalent bimatrix games have the same Nash equilibria.

(*b*) State and prove Fritz John Saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz John Saddle point theorem ? 3 + 7

3. (*a*) State and prove Slater's theorem of alternative in connection with non-linear programming.

(*b*) Minimize the following function

$$x_1x_2x_3^{-2} + 2x_1^{-1}x_2^{-1}x_3 + 5x_2 + 3x_1x_2^{-1}$$

$$x_1, x_2, x_3 > 0.$$ 4 + 6

4. (*a*) What are the differences between Beale's and Wolfe's method for solving quadratic programming problem.

(*b*) Use Beale's method to solve the following NLPP :

Minimize $Z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

subject to the constraints :

$$x_1 + x_2 \leq 2$$

and $x_1,\ x_2 \geq 0.$ 3 + 7

5. (*a*) Use the chance constrained programming technique to find an equivalent deterministic form of the following stochastic programming problem :

$$\text{Minimize } f(x) = \sum_{j=1}^{n} C_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \le b_i$$

$$x_j \ge 0, \; i. \; j = 1, 2, \ldots, n$$

where C_j is normally distributed random variable. 7

(*b*) Define the following : 3

(*i*) the primal quadratic minimization problem.

(*ii*) the quadratic dual (maximization) problem.

6. (*a*) Let θ be a numerical differentiable function on an open convex set $\mu C R^n$. Prove that θ is concave on μ iff

$$\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1)$$

for each $x^1, x^2 \in \Gamma$. 5

(*b*) State Farkas' theorem. Give the geometric interpretation of it. 5

[*Internal Assessment* : 10 Marks]

MTM-404(OM)

(*Dynamical Oceanology* - II)

Answer any **four** questions : 10 × 4

1. Approximate the momentum equation when both the Ekman number and Rossby number are small. Hence deduce the equations of geostrophic approximation. 5 + 5

2. Deduce the following equation related to the contribution of Ekman layer on a sloping surface :

$$\frac{\partial^4 v}{\partial \rho^4} + 4v = 4 \sec \theta \frac{\partial p}{\partial x'}$$

(symbols have their usual meanings). 10

3. (*a*) Show that potential vorticity is conserved in case of shallow water theory.

(*b*) Show that for large scale of motion vertical equation of motion can be written as 5 + 5

$$\frac{\partial p}{\partial z} = -\rho g.$$

4. Establish the condition for the existence of inertial boundary layer in a two-dimensional model. Show that accelerated boundary flow exists at the western shore of the ocean. 10

5. Discuss pure drift current in a sea as a result of wind stress acting on the surface of the sea, assuming that there is no pressure gradient with the water mass. 10

6. Show that the total flow function ψ is the Ekman's theory for a small Ekman number E satisfies the equation

$$\Omega E\sqrt{\sin\varphi}\,\nabla_h^2\psi + \frac{2\Omega}{a^2}\frac{\partial\psi}{\partial\lambda} = \mathrm{rot}_z(\tau)$$

where the symbols have their usual meaning. 10

7. Give a mathematical formulation of the linear model of thermocline, show that the

perturbation temperature T_s outside the Western boundary layer may be expressed in the form

$$T_s = \frac{2\theta(y)}{\pi} \int_0^\infty \frac{\sin x\tau}{\tau}\left(1 - e^{-x^2 f^2 \tau^4}\right) d\tau$$

where the symbols have their usual meaning. 10

[*Internal Assessment* : 10 Marks]
