 MSC/IIS/MATH/MA 1205/08

2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA-1205

(*Functional Analysis*)

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** questions from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *two* : 2 + 2

(*a*) Define a complete metric space.

(*b*) Define norm of a bounded linear operator.

(*c*) Give an example of a normed linear space which is not complete.

(*Turn Over*)

2. (*a*) Define a metric space. Show that the function $d: R^n \times R^n \to R$ defined by

$$d(x, y) = \sum_{i=1}^{n} |x_i - y_i|$$

is a distance function. Geometrically indicate its difference from the usual metric in R^2.

(*b*) Show that the space

$$l_p = \left\{ x = \{x_n\} : \sum_n |x_n|^p < \infty \right\}$$

is a separable metric space for a suitable metric (to be stated by you). 6 + 6

3. (*a*) Let (X_1, d_1) and (X_2, d_2) be metric spaces. Let $f: X_1 \to X_2$ be continuous and X_1 be compact. Prove that f is uniformly continuous.

(*b*) State and prove Banach – Stienhans theorem. 6 + 6

4. (*a*) If $\{e_1, e_2, \dots, e_n\}$ is a finite orthonormal set in an inner product space X and x is any element of X, then prove that

$$\sum_{i=1}^{n} |(x, e_i)|^2 \leq \|x\|^2$$

and $\left[x - \sum_{i=1}^{n} (x, e_i) e_i\right] \perp e_j$ for all $j = 1, 2, \dots, n$.

(*b*) If T is a linear transformation from a nls X into a nls Y, then prove that if T is continuous at one point of X then T is continuous at every point of X and also prove that T is bounded. 6 + 6

5. (*a*) Let X be a real normed linear space in which parallelogram law holds. Prove that X is an inner product space.

(*b*) Prove that every positive operator is self adjoint. 6 + 6

6. (*a*) Prove that under certain conditions (to be stated by you) a Fredholm integral equation

$$y(s) = x(s) - \mu \int_a^b K(s, t)\, x(t)\, dt$$

has unique solution.

(*b*) Let T be bounded linear operator of the Hilbert space X into itself and T^* be the adjoint of T. Prove that

$$\|T^*\| = \|T\| \text{ and } \|T^* T\| = \|T\|^2.$$ 6 + 6

[*Internal Assessment* - 10]
