2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(1st Semester Examination)

(*Ordinary Differential Equations and Special Functions*)

PAPER—MA-1103

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** questions from **Q. No. 2** to **Q. No. 5**

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers whenever necessary

1. Answer any *five* of the following: 2 x 5

(*a*) Let $P_n(z)$ be the Legendre polynomial of degree n and let

$$P_{m+1}(0) = -\frac{m}{m+1} P_{m-1}(0), \; m = 1, 2, \ldots$$

If $P_n(0) = -\frac{5}{16}$, then find the value of

$$\int_{-1}^{1} P_n^2(z)\, dz.$$

(*b*) Define ordinary and singular points of a second order ordinary differential equation.

(*c*) Write down the important properties of Green's function involving ODE.

(*d*) Define orthonormal functions.

(*e*) What do you mean by Bessel's functions of order n. State for what values of n the solutions are independent of Bessel's equation of order n.

(*f*) Find the indicial equation corresponding to the singularity $z = 0$ of Gauss' hypergeometric equation.

2. (*a*) Show that eigenfunctions of a regular Sturm-Liouville system

$$\frac{d}{dx}\left\{p(x)\frac{dy}{dx}\right\} + \{\lambda \varrho(x) - q(x)\} y = 0$$

having different eigenvalues are orthogonal with respect to the weight function $\varrho(x)$, where λ is a parameter, p, ϱ and q are real–valued functions of x and p, ϱ are positive.

(*b*) If $f(z)$ is continuous and has continuous derivatives in $[-1, 1]$ then prove that $f(z)$ has unique Legendre series expansion given by

$$f(z) = \sum_{n=0}^{\infty} C_n P_n(z)$$

where P_n's are Legendre polynomials and

$$C_n = \frac{2n+1}{2} \int_{-1}^{1} f(z) P_n(z)\, dz,$$

$n = 1, 2, 3, \ldots$.

5 + 5

3. (*a*) Discuss Frobenius procedure of finding the series solution about the regular singular point at the origin for an ordinary differential equation of 2nd order when the roots of the indicial equation are equal.

(*b*) Using Green's function method, solve the equation

$$\frac{d^2y}{dx^2} = f(x)$$

subject to $y(0) = y(a) = 0$. 5 + 5

4. (*a*) Solve the system of differential equation

$$\frac{dx}{dt} = Ax$$

where $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 1 \end{bmatrix}$ and

$$x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

(*b*) Find the solution of the Gauss' hypergeometric equation

$$z(1-z)\frac{d^2w}{dz^2} + \{c-(a+b+1)z\}\frac{dw}{dz} - abw = 0$$

near the point at infinity, where a, b, c are constants. 5 + 5

5. (*a*) Find the Legendre polynomial $P_n(z)$ by solving the following differential equation

$$(1-z^2)\frac{d^2w}{dz^2} - 2z\frac{dw}{dz} + n(n+1)w = 0.$$

(*b*) Show that

$$\sqrt{\frac{\pi z}{2}}\, J_{3/2}(z) = \frac{1}{z}\sin z - \cos z.$$

5 + 5

[*Internal Assessment* — 10 Marks]