2008

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MA 2106

Full Marks : 50

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(*Dynamical Meteorology* -I)

Answer **Q.No.1** and any *four* from the rest

1. Answer any *two* questions : 2x2

(*a*) What do you mean by adiabatic process ?

(*b*) Define wet bulb temperature.

(*c*) What is the purpose of Acrological diagrams ?

2. (*a*) Show that the sum of kinetic energy, potential energy and enthalpy of an air parcel is constant when the flow is steady, adiabatic and friction-less.

   (*b*) Derive the specific heat constant with respect to pressure of moist air in terms of dry air. 6 + 3

3. (*a*) Show that the relative increase in dew-point temperature is about 5% of the sum of the relative increases in mixing ratio and pressure during isobaric cooling of air parcel.

   (*b*) What is the concept of Geo-dynamical paradox in the atmosphere ? Hence derive the equation of Geostrophic wind. 4 + 5

4. (*a*) Derive the equation of momentum of an air parcel in cartesian co-ordinates.

   (*b*) Define pseudo-wet bulb temperature and pseudo-potential temperature. 7 + 2

5. (*a*) Derive the saturated adiabatic lapse rate of moist air and hence show that it is less than dry adiabatic lapse rate.

   (*b*) Explain the convergence and divergence in the atmosphere. 6 + 3

6. (*a*) Derive the area equivalence of the Tephigram and discuss its important features.

(*b*) Define Barotropic and Baroclinic atmosphere. 7 + 2

7. (*a*) Derive an expression for the density ϱ that results when an air parcel initially at pressure p_s and density ϱ_s expands adiabatically to pressure p.

(*b*) State and prove the hypsometric formula. 4 + 5

8. (*a*) Define homogeneous atmosphere. Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of homogeneous isothermal atmosphere is equal to $\frac{1}{e}$ times that at the sea level.

(*b*) What is the concept of coriolis force in the atmosphere? (1 + 3 + 2) + 3

[*Internal Assessment* : 10 Marks]

(*Operational Research Modelling* -I)

1. Answer any *four* questions : 2×4

(*a*) State Bellman's principle of optimality in dynamic programming.

(*b*) What are the differences between CPM and PERT ?

(*c*) What are the advantages to use simulation to solve a problem ?

(*d*) What do you mean by 'EOQ' model ? Derive the optimum order quantity for EOQ model stating the assumptions.

(*e*) Explain the 'Individual' and 'Group' Replecement policies. Give the expression of average failure following Mortality theorem in the case of Individual Replacement Policy.

(*f*) Explain the nature of a queueing system represented by $(a/b/c:d/e)$. Give the expression of $\lambda_{effective}$ in the case of a finite queue.

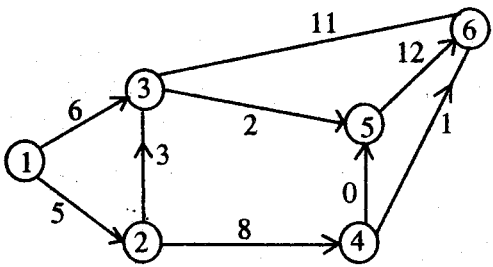
2. Answer any *four* questions : 8 x 4

(*a*) Solve the following problem by dynamic programming method :

$$\text{Minimize } Z = y_1^2 + y_2^2 + y_3^2$$

$$\text{subject to } y_1 y_2 y_3 \leqslant 4,$$
$$y_1,\ y_2,\ y_3 \geqslant 0.$$

(*b*) Determine the earliest and latest occurrence time for the following project network, all the duration are in days. Determine the time schedule and the path for the following project.



(*c*) Explain Monte Carlo simulation to evaluate the value of π. Also, explain the limitations of this method.

(*d*) Find the optimum order quantity for a product with the following price breaks :

Quantity	Pruchasing price (Rs.) per unit
$0 < Q_1 < 100$	20
$100 \leqslant Q_2 < 200$	18
$200 \leqslant Q_3$	16

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product and the cost of ordering is Rs. 25·00 per month.

(*e*) At time zero, all the items in a system are new. Each item has a probability p of failing immediately before the end of first month of life and probability $q\,(=1-p)$ of failing immediately before the end of the second month. If all items are replaced as they fail, show that the expected number of failures, $f(x)$ at the end of x months is given by

$$f(x) = \frac{N}{1+q}\left[1 - (-q)^{x+1}\right],$$

N = No. of items in the system.

If the cost per item of individual replacement is C_1 and the cost per item of group replacement is C_2. Find the condition under which a group replacement policy at the end of each month is most profitable.

(*f*) In a car washing service facility information gathered indicates that cans arrive for service according to a Poisson distribution with mean 5 per hour, the time for washing and cleaning each car varies but is found to follow an exponential distribution with mean 10 min. per car. The facility cannot handle more than one car at a time and has a total of 5 parking spaces. If the parking space is full, newly arriving cars balk away to seek service elsewhere.

(*i*) Find the average number of cars in the system.

(*ii*) Find the mean waiting time for a car to be serviced.

(*iii*) How many car balk away in a day (assume 8 working hour per day).

(*g*) Find the optimum order level which minimize the total expected cost under the following assumptions :

(*i*) t is the constant interval between order.

(*ii*) Q is the stock (in continuous units) at the beginning.

(*iii*) r is the estimated random instantaneous demand at a discrete rate.

(*iv*) C_1 and C_2 are the holding and shortage costs per item per t time period.

(*v*) Lead time is negligible.

[*Internal Assessment* : 10 Marks]