New

2017

BCA 1st Semester Examination

DISCRETE MATHEMATICS

PAPER—1103

Full Marks : 70

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer Q. No. *1* and any *six* questions from the rest.

1. Answer any *five* questions : 5×2

(a) Define partial order set with an example.

(b) If $f : R \to R$ is defined by $f(x) = |x| + x$, $x \in R$ and $g : R \to R$ is defined by $g(x) = |x| - x$, $x \in R$ then find f_0g.

(c) Define integral domain.

(d) Make a truth table for $a \leftrightarrow b$.

(Turn Over)

(e) Define a spanning tree with an example.

(f) If $a \equiv x \pmod{n}$ and $b \equiv y \pmod{n}$, then show that $ab \equiv xy \pmod{n}$.

(g) A sequence is defined recursively by $a_0 = 2$, $a_1 = 3$ and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$. Find a_4 and a_5.

(h) How many committees of five people can be choosen from 12 men and 20 women it at least three women must be on each committee ?

2. (a) If A, B, C are subsets of the universal set s, then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. 5

(b) A relation ρ on Z (set of all integers) is defined as $\rho = \{(a, b) \in Z \times Z : 3a + 4b\}$ is divisible by 7. Examine whether ρ is an equivalence relation or not. 5

3. (a) Examine the mapping $f : Z \rightarrow Z$ defined by $f(x) = 2x + 5$, $x \in Z$ is bijective or not. 5

(b) Let $A = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $a \leq b$ if a devides b. Draw the Hasse diagram of $(A, \leq)$. 5

4. (a) Use mathematical induction to prove that $16^n + 10n - 1$ is divisible by 25 for all $n \geq 1$. 5

(b) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$, $n \geq 2$, with initial conditions $a_0 = 1$, $a_1 = 4$. 5

5. (a) Use truth table to show that
$[(p \vee q) \vee ((q \vee (\neg r)) \wedge (p \vee r))] \leftrightarrow \neg [(\neg p) \wedge (\neg q)]$. 5

(b) Show that Z_5, the classes of residues of integers modulo 5, forms an abelian group with respect to +, addition (modulo 5). 5

6. (a) In a group (G, 0), $(a_0 b)^2 = a^2 {}_0 b^2$ holds for all a, b $\in$ G. Prove that the group is abelian. 5

(b) Prove that the ring of matrices $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b, \in R \right\}$ is a field. 5

7. (a) Draw a graph with the help of Adjacency matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$ 3

(b) In a ring (R, +, .), prove that a.0 = 0.a = 0. 2

(c) Find the number of combinations that can be obtained by letters of the word COMMERCE taking 4 at a time. 5

8. (a) Prove that for a simple graph with n vertices and m components can have at most (n − m) (n − m + 1)/2 edges. 5

(b) In a class of 25 students, 12 have taken economics, 8 have taken economics but not political science. Find the number of students who have taken economics and political science and those who have taken political science but not economics. 5

9. (a) Prove that a connected graph with n vertices is a tree if and only if it has n − 1 edges. 6

(b) Define a Hamiltonian cycle. Give two examples in which one is Hamiltonian and another is not Hamiltonian. 4

10. (a) A Boolean function f is defined by f(x, y, z) = xy, yz, zx. Find the conjunctive normal form of f(x, y, z). 5

(b) Simplify the Boolean expression

$$\bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 x_2 x_3$$ using K-map. 5

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