NEW

2017

BCA

2nd Semester Examination

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

PAPER—1203

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer *Q. No. 1* and any *six* from the rest taking at least *one* from each group.

1. Answer any *five* questions : 5×2

(i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$.

(Turn Over)

(ii) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, prove that $A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(iii) Find the product of the eigen values of the matrix

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(iv) If the event A and B are independent, then show that A^c and B^c are independent.

(v) State Cauchy mean value theorem.

(vi) If $y = e^{ax} \sin bx$, prove that

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)\, y = 0.$$

(vii) Two coins are thrown simultaneously. Find the probability of getting at most one head.

(viii) Write the properties of correlation coefficient.

(ix) Define homogeneous function of several variables and state Euler's theorem on it.

Group—A

(Algebra)

2. (a) Find the equation whose roots are the roots of $x^3 - 6x^2 + 11x - 6 = 0$ each increased by 1. 5

(b) Solve the equations
$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$
by Cramer's rule. 5

3. (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$. 6

(b) Using Cayley-Hamilton theorem find A^4, if $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$. 4

4. (a) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)$. 5

(b) Show that the mapping $f: R \to R$ defined by $f(x) = |x| + x$ is neither one-one nor onto. 5

Group—B

(Calculus)

5. (a) (i) State Lagrange's mean value theorem.

 (ii) State Taylor's theorem with Lagrange's form of remainder. 5

 (b) If $u = \tan^{-1}\frac{y}{x}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 5

6. (a) Evaluate $\lim_{n\to\infty}\left[\frac{n}{n^2} + \frac{n}{l^2+n^2} + \frac{n}{2^2+n^2} + \dots + \frac{n}{(n-1)^2+n^2}\right]$. 5

 (b) Evaluate $\int \frac{e^{-x}dx}{e^x + 2e^{-x} + 3}$. 5

7. (a) If $y = a\cos(\log x) + b\sin(\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)yn = 0$. 5

 (b) Answer any *one* question : 5

 (i) Evaluate $\int_0^{1\frac{1}{4}} \log(1+\tan\theta)\, d\theta$.

 (ii) Evaluate $\int_0^{1\frac{1}{2}} \frac{dx}{5+3\cos x}$.

Group—C

(Probabilities)

8. (a) If $P(A)=\frac{3}{8}$, $P(B)=\frac{1}{2}$, $P(A\cap B)=\frac{1}{4}$ then find $P(\bar{A}/\bar{B})$ and $P(\bar{B}/\bar{A})$. 5

(b) Explain Skewness with geometrical concept. 5

9. (a) State and prove Bayes' theorem of probability. 5

(b) Is the function defined as follows a density function ?

$$f(x)=\begin{cases} e^{-x}; x\geq 0 \\ 0\ ;\ x<0 \end{cases}$$

If so, determine the probability that the variate having this density will full in the interval (1,2) ? 5

10. (a) Calculate the median and mode of the following frequency distribution 5

Height in inches :	56-60	61-65	66-70	71-75	76-80
No. of persons :	7	25	43	28	7

5

(b) Find the mean and variance of continuous distribution given by the following density function

$$f(x) = \frac{1}{2} - ax \text{ if } 0 < x < 4$$
$$= 0 \text{ elsewhere.}$$ 5

[Internal Assessment — 30 Marks]
