2017

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Rigid Dynamics*)

[*Marks* : 30]

1. Answer any *three* questions : 8×3

(*Turn Over*)

(*a*) Define degrees of freedom. Show that the rate of change of angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body.

(*b*) Define principle axes. Find the conditions that y-axis for given material system to be a principal axis at any point of its length ; also find the other two principal axes. What happens when an axis passes through the C.G. of the body ? Justify.

(*c*) A lamina, in the form of an ellipse, is rotating in its own plane about one of its foci with angular velocity w. The focus is set free and the other focus at the same instant is fixed. Show that the ellipse now rotates about it with angular velocity $\frac{2-5e^2}{2+3e^2}w$, where e is the eccentricity of the ellipse.

(*d*) Find the time period of a compound pendulum. A bent lever, whose arms are of length a and $2a$, the angle between them being 60°, makes small oscillation in its own plane about fulcrum ; find the length of the corresponding simple pendulum.

(*e*) Two uniform sphers, each of mass M and radius 'a' are firmly fixed to the ends of two uniform thin rods, each of mass m and length l, and the other ends of the rods are freely hinged to a point O. The whole system revolves, as in the governor of a steam engine, about a vertical line thorugh O with angular velocity ω. Show that when the motion is steady, the rods are inclined to the vertical at an angle θ given by the equation :

$$\cos\theta = \frac{g}{\omega^2} \cdot \frac{6M(l+a)+3ml}{6M(l+a)^2+2ml^2}.$$

2. Answer any *two* questions : 3×2

(*a*) State and prove the law of conservation of linear momentum under impulsive forces. $1 + 2$

(*b*) Deduce the general equations of motion in two dimensions in case of varying mass of the rigid body. 3

(*c*) Calculate the product of inertia of a semi-circular wire about diameter and tangent at its extremity. 3

GROUP – B

(*Hydrostatics*)

[*Marks* : 25]

3. Answer any *two* questions : 8×2

(*a*) Define a metacentre. Find the condition of existence of a metacentre of a body. Obtain the formula for finding the metacentre of a body following freely in a homogeneous liquid at rest under gravity.

(*b*) Find the centre of pressure of a triangular area when (*i*) one vertex is on the free surfae and (*ii*) depths of vertices are given.

(*c*) A homogeneous right circular cone of vertical angle 2α floats with its axis vertical and vertex downwards and is in unstable equilibrium. Prove that the position becomes stable when the particle of weight w is attached to the vertex if

$$\frac{w}{W} > \left\{ \left(\frac{e \cos^6 \alpha}{\sigma} \right)^{\frac{1}{4}} - 1 \right\}$$

where W is the weight of the cone. It is assumed that the base of the cone is not submerged, σ the density of the cone ρ the density of the liquid.

4. Answer any *three* questions : 3×3

(*a*) Find the surface of equidensity and equipressure when the force field is conservative.

(*b*) A closed circular cylinder is just filled with water and rotates about its axis which is vertical ; find the pressure at any point in the fluid.

(*c*) Show that the equation for a gas in an adiabatic temperature changes is $TV^{\gamma-1} = \text{const}$, the symbols having usual meanings.

(*d*) A right circular cylinder of specific gravity (sp. gr.) ρ floats in water with its axis vertical, two-thirds being immersed in water. If σ be the sp. gr. of air, the prove that $3\rho = 2 + \sigma$.

(*e*) If the density of a liquid at rest under gravity varies as the square root of the presure then prove that the density increases uniformly with the depth.

GROUP – C

(*Discrete Mathematics*)

[*Marks* : 20]

5. Answer any *one* question : 15 × 1

(*a*) (*i*) Define bounded lattice with an example. Prove that every chain is a distribution lattice. 1 + 1 + 3

(*ii*) Define generating function of a numerical function. Find the sequence represented by the closed form of the generating function

$$f(x)=\frac{1}{(1-x)^2},\ |x|<1.$$ 1 + 5

(*iii*) What do you mean by Tautology and Contradictions ? Give one example in each. 2 + 2

(*b*) (*i*) Show that any graph is connected if it has atleast $(n\text{-}1)$ edges, where n is the number of vertices in the graph. 5

(*ii*) State the necessary and sufficient condition for a bipartile graph. Define bipartite isomorphism of graphs. 4

(*iii*) Define poset. If n be a positive integer and D_n denotes the set of all divisors of n, consider the partial order 'divides' in D_n. Then draw the Hasse diagrams for D_6, D_{24}, D_{30}. 1 + 5

6. Answer any *one* question : 3 × 1

(*a*) Find the time complexity of linear search algorithm.

(*b*) Prove that the set D of all factors of 12 under divisibility forms a lattice.

7. Answer any *one* question : 2 × 1

(*a*) Define spanning tree. Is every graph has spanning tree ? Is it unique ? Explain.

(*b*) State the principle of inclusion and exclusion.

GROUP – D

(*Mathematical Modelling*)

[*Marks* : 15]

8. Answer any *one* question : 15 × 1

(*a*) (*i*) Find the phase path and draw it on the phase diagram

$$\frac{dx}{dt} = x + y, \ \frac{dy}{dt} = x - y.$$ 5

(*ii*) State the assumptions and formulate the simple epidemic (SI) model. 2 + 3

(*iii*) Show that in case of SI model, ultimately all the person's will be infected. 5

(*b*) (*i*) Discuss Age-structured population model with position of equilibrium. 7

(*ii*) If the model equation of the growth of a single species is

$$\frac{dx}{dt} = 0 \cdot 71\left(1 - \frac{x}{80 \cdot 5 \times 10^6}\right)x$$

with $x(0) = x_0 = 20 \cdot 125 \times 10^6$. Find population at any time t. Also find t when $x(t) = 60 \cdot 375 \times 10^6$ kg. 8